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ABSTRACT

This report describes a program for improving elementary school students ability to solve mathematical word problems. The targeted population consisted of primary, middle, and junior high students attending two different kindergarten through eighth grade schools. Both schools were located within blue collar neighborhoods in a large metropolitan area. Students' weakness in the area of problem solving was documented by teacher-made tests, journaling, standardized test scores, student surveys, and teacher surveys. Research literature and measurement tools revealed the following probable causes: inability to read story problems adequately; poor reading ability in general; improper strategy use; lack of strategy use; lack of desire to properly understand mathematical logic of problem; strategies that rely on memorization; insufficient instructional time spent on problem solving; and inadequate time spent on finding solutions. A review of solution strategies suggested by knowledgeable others, combined with an analysis of the problem setting, resulted in the selection of seven major categories of intervention: problem of the day, cooperative grouping, pair/sharing, illustrating problem data, math journaling, classification of word problems, and use of analytical worksheets. Post intervention data indicated an increase of strategy use, a positive change in student attitude towards word problems, and an increased in time spent on each problem solution. (Contains 30 references.) (Author/WRM)

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MATH WORD PROBLEM REMEDIATION WITH ELEMENTARY STUDENTS

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ABSTRACT

This report describes a program for improving elementary school students ability to solve mathematical word problems. The targeted population consisted of primary, middle, and junior high students attending two different kindergarten through eighth grade schools. Both schools were located within blue collar neighborhoods in a large metropolitan area. Students' weakness in the area of problem solving was documented by teacher-made tests, journaling, standardized test scores, student surveys, and teacher surveys.

Research literature and measurement tools revealed the following probable causes: inability to read story problems adequately; poor reading ability in general; improper strategy use; lack of strategy use; lack of desire to properly understand mathematical logic of problem; strategies that rely on memorization; insufficient instructional time spent on problem solving; and inadequate time spent on finding solutions.

A review of solution strategies suggested by knowledgeable others, combined with an analysis of the problem setting, resulted in the selection of seven major categories of intervention: problem of the day, cooperative grouping, pair/sharing, illustrating problem data, math journaling, classification of word problems, and use of analytical worksheets.

Post intervention data indicated an increase of strategy use, a positive change in student attitude towards word problems, and an increased in time spent on each problem solution.

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CHAPTER 1

PROBLEM STATEMENT AND CONTEXT

General Statement of the Problem

A major challenge elementary school teachers face is to successfully teach and motivate students to utilize math word problem strategies. Evidence of a need for improvement in this area are poor student attitudes regarding word problems and low standardized test scores on these types of problems. Five classrooms at two sites have been the focal point of this study. Site A includes three targeted classrooms; a third grade self-contained classroom and a sixth grade self-contained classroom, as well as an eighth grade departmental classroom. Site B includes a fourth/fifth grade self-contained split classroom and an eighth grade departmental classroom.

School and Community Descriptions

The targeted classrooms discussed at this site will be referred to as classrooms one, two, and three. The first site A researcher teaches in a third grade classroom. The second site A researcher teaches in a sixth grade classroom. The third site A researcher teaches in an eighth grade classroom. The first and second classrooms are self-contained and the third classroom is departmental Social Studies. Included in the weekly schedule for the researchers are three, 40-minute planning periods. Students take library, music, and gym classes during these times. All classrooms receive 20 minutes for lunch per day.

Classroom one is located within a four-room mobile unit that is independent of the main school structure. The class consists of 30 students who range in age from eight to nine. 17 students in the targeted classroom are male and 13 are female. Twenty-four students receive free or reduced lunches due to economic need. One student has been classified as having learning disabilities. One of the students in the targeted classroom is repeating the third grade.

Classroom two is located within a 12-room annex that is also independent of the main school structure. The class consists of 34 students who range in age from 11-13. Twenty-three students in the targeted classroom are male and 11 are female. Thirty students receive free or reduced lunches based on economic need. One student receives bus services to and from school. Six of the students have been classified as having learning disabilities, two students receive bilingual services, and three attend speech classes.

Classroom three is located in the main school structure. The class consists of 28 students who range in age from 13-15. Fifteen students in the targeted classroom are male and 13 are female. Twenty students receive free or reduced lunches due to economic need. Two of the students have been classified as having learning disabilities, two students receive bilingual services, and two attend speech classes.

Site A was built in 1926 as a single structured K-8 Devor style public school. The campus covers half a square city block in area. Surveillance cameras and high powered lights have been added outside the school to protect against graffiti and loitering. The dramatic population increase within the targeted school's attendance area has necessitated the construction of a 12-room annex as well as the addition of three, four- room mobile units.

The main structure consists of 30 classrooms, 16 of which are on the first floor and 14 on the second floor. All rooms in all buildings are equipped with one new computer. The first floor consists of a main office, auditorium, gymnasium, library, two reading resource rooms, a speech resource room, 10 regular classrooms from grades kindergarten through two, a teachers' lounge, and four washrooms. The first floor includes framed portraits of the past 18 graduating classes as well as famous prints from the Art Institute of Chicago. The second floor consists of a support staff office, two bilingual resource rooms, a reading/math resource room, 10 regular classrooms including grades three, seven, and eight, the counselor's office, school cafeteria, and four washrooms. The second floor is decorated with student work monthly by alternating staff members.

The newly constructed annex contains an office, 12 classrooms from grades four, five, and six, two janitorial supply rooms, and two washrooms. Site A is currently in the process of wiring all classrooms in the annex for internet access. All classes in the annex have lunches delivered to them to avoid overcrowding the school's cafeteria, which was originally designed for a school of 500 students. In addition, there are three mobile units which each contain four classrooms. One of the mobile units contains usable student washrooms. The first mobile unit consists of grades three and four. Another mobile unit contains grades two, five, and six. The last mobile unit contains rooms for the school psychologist, social worker, nurse, and special education teachers. All mobile units are surrounded by flower boxes and potted bushes.

Site A consists of 1207 students and has an average class size of 30 students. Of the enrolled students, 71% are of Hispanic descent, 27% are White, and African Americans and Asian/Pacific Islanders each account for one percent. Approximately 34% of the students are classified as having limited English proficiency. There are approximately 250 students enrolled in

the Spanish bilingual program and another 60 in the Polish bilingual program offered at Site A. Low income students comprise 74% of the student population and are eligible for free or reduced lunches. Site A has a 96% student attendance rate and a student mobility rate of 21%. According to the School Report Card, there are no chronic truancies at Site A.

Site A has a dedicated staff of 75 employees. Of these, 61 are teachers whose average teaching experience is 15 years. Forty-one percent of the teachers have educational backgrounds of Master's Degrees or above. The average teacher salary for 1997 was $40,505. The racial makeup of the staff is 51% minority and 49% White, Arab, or Asian. The attendance rate for the teaching staff in 1997 was 95% according to the School Report Card.

There are nine career service personnel employees at Site A. Included in this group are a bilingual aid, three school assistants, two teacher assistants, a child welfare attendant, and two clerks. School aids and assistants often lead small-group reading sessions and are given other duties by the assistant principal. In addition, Site A employs 16 safety patrol members who serve as crossing guards, monitors, and parent tutors.

The administration at Site A is a highly motivated team comprised of a male principal and two female assistant principals. All three administrators have been at Site A for the past five years and in the education field for at least 15 years. The average administrator salary at Site A for 1997 was $66,100.

There are many extracurricular activities for students at Site A. In order to improve students' reading and math skills, after school tutoring programs have been implemented. Students two grades below grade level in either reading or math work on basic skills twice a week with staff members. The student to teacher ratio is approximately 10:1. There is also a computer club,

student peer program, writing contests, school newspaper club, student council, and a Great Books program. Site A also has both a volleyball and basketball team. There is also a social center program which offers basketball, table games, movies, art, and homework help sessions twice a week after school. Parent volunteers help make this program possible. Site A is located within Region Five of the school system, which coordinates with the extracurricular and academic events within the scheduled school year.

The community to which Site A belongs is a blue collar, working class neighborhood on the southwest side of a major city. Transportation to and from the downtown area is easily accessible by public bus or train. The neighborhood is near many major intersections and a major expressway. It is located one mile from a major airport and less than a mile from a major healthcare center. There is a public library within one half mile of the school. Also, to relieve overcrowding in many local schools, there is a new public school currently under construction in the area.

The community is composed mostly of well kept single-family homes and apartments. The neighborhood is well established with older homes having an average cost of $110,000. As of 1990, 82% of the homes were owner occupied. In addition, 85% of the area was White, 13% was Hispanic, with 1% African American and 1% Asian. The median family income for the area in 1990 was $37,400. The population in the area has steadily increased in the past eight years due the increase in multi-family residences. Its racial makeup has also changed dramatically in the past eight years, with Mexicans making the most dramatic increase at eight percent. The area is a rapidly aging one—almost one fourth of its residents are at least 65 years old. These statistics have been gathered from the School Report Card.

Site B is a K – 8 school located in a major metropolitan area of the Midwest. The targeted classrooms discussed at this site will be referred to as classrooms one and two.

Classroom one is a self-contained fourth fifth grade split class of 32 students. The make up of the class consists of approximately 90% neighborhood children; the remaining students are options children bussed in from other school populations (options program will be explained further into the site description.) The school day begins at 9:05 AM and ends at 2:30 PM, with a 20-minute lunch occurring at 11:30 AM. Within the school week, students also attend Music, Computer Lab, Library, Gym, and Art. This time is used as preparatory time for the classroom teacher.

Classroom two is a departmental room in which mathematics is taught the first four periods each day to seventh and eighth graders. In addition to mathematics instruction, the targeted classroom is taught spelling and health in the afternoon. Departmental classrooms also receive five 40-minute periods consisting of the same aforementioned special classes which allow the classroom teachers preparatory time. Furthermore, the researcher in the classroom is involved in some extracurricular mathematics instruction which affect the length of some of the school days. On Tuesdays and Thursdays a Pre-Algebra class is taught to eighth grade students for one period in the afternoon.

Site B is a two story brick building built in 1936. It is located in a blue collar, working class neighborhood. Several blocks away are major traffic arteries providing public transportation to a large metropolitan area. In 1953, an addition was built consisting of eight classrooms, bringing the total number of classrooms to 32. Just inside the entrance of the building is a mural with a safari theme that was painted by the art club. The building is in the process of being repainted by a city-wide volunteer organization. A comprehensive landscaping renovation was recently completed

which includes a playground, basketball and volleyball courts, and a new parking lot. The population increase of school age children has forced the addition of two double-mobile units placed in the rear of the main building in the summer of 1997. Each mobile unit consists of two, full-size classrooms and a restroom. Most classes need to walk back and forth to the mobiles at least once a week for the Art class. There is an intercom system which was installed in the main entrance and rear door to provide building security.

The main building has a total number of 28 classrooms. The first floor has one room each of grades K-3. In addition, there are two special education classes, which consist of a primary grade of severely learning disabled students and a teachable mentally handicapped student. There is one classroom that is shared by the math resource and special education teachers. Furthermore, there is a room devoted to English as a second language. There are also three offices on this floor; the main office, the assistant principal's office, and an office shared by the counselor, social worker, nurse, and psychologist. The school cafeteria and the teacher's lounge are on the first floor as well. Lastly, there is a gym/auditorium used for all school functions.

The second floor functions mainly as classroom space. There are three special education rooms, which consist of upper and middle grade severely mentally handicapped and teachable mentally handicapped students. There is one room each of fourth and fifth grade. In addition, there are two of each sixth, seventh, and eighth grades. Furthermore, we are fortunate to have a federal desegregation program that pays for a computer and science lab. At the end of the hall is a small library which has a walk-in closet that is used by the speech pathologist team.

Site B consists of approximately 470 students with an average class size of 32. Site B is an integrated school consisting of 65% Caucasian students, 18% African American students, 15%

Hispanic students, and two percent other. Most minority children attend because of the Options for Knowledge Program. This program enables the school to bus in children from lower income neighborhoods. Site B has 50 children in the Polish Bilingual Program, 20 children who are classified as Teachable Mentally Handicapped, and 35 children who have been classified with Learning Disabilities. Low income students comprise 54% of the student body and are eligible for free or reduced lunches and breakfast, as compared to the 36% stayed average. A large segment of the school population has Polish ancestry; eight percent are classified as limited English proficient. Site B has a 95% attendance rate, a nine percent mobility rate, and no chronic truancies according to the School Report Card.

Site B has a large and dedicated staff of 60 employees. There are a total of 30 teachers, with an average teaching experience of 18 years, and average salary of $40,500.00. The ratio of female teachers to male teachers is 27:3. Ethnic background is as follows: 22 Caucasian, seven African American, and one Hispanic. One teacher has a Doctorate degree, while ten teachers have Master's degrees. Attendance rate for teachers is 94% according to the School Report Card.

Career service personnel play an important role in maintaining a high quality of instruction. The total number of career service staff is 17 females. Ethnic background is as follows: 11 Caucasian, four African-American, and two Hispanic. The average years of experience is seven and the mean salary is $18,000.00. Some of the career service people are assigned to classrooms as permanent teacher assistants, others are used at the discretion of the administration.

Our administration is a highly motivated team who work hard to keep the school standards high. There is a male principal and a female, full-time assistant principal. The average salary of administrators is $66,000,00. Relationships among administration, community, staff, and Local

School Council are based on a sincere desire to improve the school. Many community residents and parents of Site B school are alumni, and this factor further promotes cooperation. This cooperation is evidenced by an active and supportive Parent Teacher Council.

Two features of the Site B school that are attractive to students are the computer lab and science lab. However, equipment in the computer lab has become seriously outdated. A committee of teachers is currently writing grant proposals and seeking funds to remedy this short coming. New computers and printers for some classrooms have been purchased and the focus will be to continue to equip classrooms before the computer lab is updated. The science lab is a wonderful learning environment for children. There are about fifty animals which provide many opportunities for hands-on experience.

The newest academic program is Lighthouse. This program enables low-achieving students to receive remedial instruction after school, three days a week. Teachers spend one hour focusing on math and reading. After the academic hour the children are able to attend the Social Center session. The Social Center involves several extra-curricular activities including table games, drama, and dancing. Other activities that take place before and after school are cheerleading, basketball, and volleyball.

Many extra-curricular activities are also sponsored throughout the school day. These activities are Art Club, school newspaper, yearbook, Great Books Program, Student Council, Young Authors, Builders Club, Rainbows, Junior Achievement and DARE Program. Site B is located in Region Four of the school system which coordinates with the extracurricular and academic events as scheduled within the school year.

Site B community is a largely residential neighborhood situated on the southwest border of a major city. Transportation to and from the downtown area is easily accessible by public bus and train. Furthermore, a major expressway runs through the area. These factors allow convenient travel to and from the city for purposes of employment, entertainment, and cultural events.

This community is composed mostly of well kept single-family homes and apartments. The neighborhood is well established with older homes having an average cost of $140,000.00. Very little new construction of homes is taking place due to the fact that only three percent of the land remains vacant. A high proportion (83%) of the homes are owner-occupied.

Population of this area peaked in the 1970's at 40,000 and has been gradually declining to less than 34,000 according to the 1990 Census. This community is 80% Caucasian, 13% African American, and 7% Hispanic. Approximately one third of the Caucasian community is Polish. There were no African-Americans living in this area until in the 1950's, when the city housing authority built two-story apartment houses in the northeast sector of the area. The African-American population continues to reside in this area.

This is a basically working class neighborhood with a median income from the 1990 Census of $38,000. A little more than half the careers are classified as white collar occupations. For example, many city workers such as policeman and firefighters reside here. Due to the many manufacturing plants in the nearby communities, about one fifth of the working population is employed in this field. Additionally, a major metropolitan airport is located nearby this site. Local restaurants, banks, hotels, and florists benefit from the airport's presence. A major public transportation route connecting to downtown bisects the community and is one city block from Site B. On this busy street are many retail businesses and restaurants.

National Literature Citations

Students have difficulty motivating themselves to solve word problems, and therefore experience difficulty in this area of math (Verzoni, 1997). Given this observation, one has to consider the reasons and solutions for this ongoing problem. Leblanc and Russell state that text wording and the interaction of text with mathematical problems hinders the understanding of the concepts contained within word problems (1996). Teachers need to address reading skills in terms of vocabulary and recurrent word patterns when teaching math. In addition, literature itself should be utilized in order to critically analyze situations within a context that can be associated with math word problems (Smith, 1995).

Another reason children experience difficulty with word problems is due to incorrect mental reasoning. Therefore, demonstrating and explaining the proper strategy is not sufficient: the teacher must relate instruction to the students incorrect reasoning (Campbell, 1997). Cooperative groups, pair sharing, and manipulatives are singled out as both motivating and integral concept builders that assist in developing "real world" reasoning abilities. Therefore, it is vital that teachers provide students with the means of transferring information within a word problem to a situation they may encounter outside the school surrounding.

The theme of relating word problems and tasks associated with them to "real life" are recurrent throughout the literature as a solution to aiding student word problem solving abilities. Students who can relate a given word problem to a situation familiar to them have a concrete basis for understanding the steps involved to arrive at the solution. In addition, students are able to utilize strategies with more confidence when they can relate the scenario of the problem to prior

knowledge. Some solutions include: drawing pictures of the problem, using travel experiences to discuss math, and using imagery techniques when solving word problems. The following chapters of this paper will discuss, in detail, the findings related to each of these techniques and strategies, based on their development and implementation.

CHAPTER 2

PROBLEM DOCUMENTATION

Review of Literature for Probable Causes

A major challenge middle and upper grade teachers face is to successfully teach and motivate students to utilize math word problem strategies. Solving word problems is one of the most difficult tasks for students, as stated by Potter, Whimbley, and Lochhead (1991). As Desmond Morris states in *The Naked Ape,* man has been struggling with problem solving since climatic changes caused him to abandon his treelike habitat and compete with the better equipped predators on the ground. Indeed, based on Morris' findings, man would have ceased to exist if he did not rely on his problem solving strategies (1979). If we accept the premise that man and his accomplishments are based upon his problem solving abilities, then it follows that he must not only retain such abilities but sharpen them as well, if we are to continue as a viable species. In this light, learning to problem solve becomes grandiose in nature rather than being simply one facet of math.

A study by Mullis (1992) found that students generally perform better at computational activities that require little or no higher-order thinking skills than they do on word problems. Within the study only 16% of fourth graders and 8% of eighth graders were able to successfully explain their answers or provide a reasonable strategy through which to solve math word problems. Teachers should see these statistics as an alarm on the educational clock to "wake" them up. Additional evidence for a need to improve in this area is poor student attitudes regarding word problems and low standardized test scores on these types of problems. This paper will attempt to identify the causes of student apathy and ineptitude regarding math word problem solving.

The research literature referenced below centered on four basic causes for student lack of achievement in problem solving. First, reading ability was closely tied to student failure. Second, there is much evidence that children do not make use of systematic strategies when presented with problems. Third, many students focus on answer attaining and not on understanding of the solution. Finally, there is evidence that most students do not allow themselves sufficient time to properly work out the problems.

A major cause of student failure to solve word problems is their inability to read the problem correctly. Most math teachers naturally recognize the connection between reading ability and success in working story problems. Low reading ability students are usually baffled by word problems. LeBlanc (1996) describes how poor readers' problems are further exasperated by the fact that the texts themselves have wording that is inherently confusing. Some educators find themselves addressing this problem by reminding their students to "read the problem carefully" However, this is an unproductive teaching strategy since students do not have a clear idea of what reading a story problem "carefully" means (Parmar and Crawley, 1994). As Cassidy (1991) states, reading math problems requires different reading skills than students may employ in other content areas. A poor reader will focus on skimming over the problem and seeking an easy answer. They do not read the material thoroughly enough to understand the question being asked. Also, this quick skimming prevents students from recognizing which facts are necessary to the solution (Potter, Whimbley, Lochhead, 1991).

Another major cause of problem solving failure among students is that they do not use a correct approach when finding the answers. When attacking a problem, students often do not use any method or logical sequence of steps (Potter, Whimbley, Lochhead, 1991). Secondly, children sometimes have their own personal mental strategy which has an internal logic to it, but is incorrect (Campbell, 1997). If the teacher merely demonstrates the correct method or explains how to solve

the problem, the child will still not comprehend the solution. Their incorrect internal logic needs to be addressed. However, in this scenario, discovering and correcting the child's misconceptions is a difficult task for the teacher.

A third major cause of student failure is that students attempt to memorize a strategy and blindly apply it without seeking to truly understand the problem (Miller, 1996). This problem is often compounded by educators' attempts to "teach" story problem "rules", such as looking for clue words as "altogether" or "left" (Parmar & Cawley, 1994). Children rely on these instructional crutches and never develop an awareness of mathematical solutions. In fact, Potter, Whimbey, and Lockhead (1991) found that memorizing cue words actually prevented students from developing a conceptualization of the problem as they concentrated on looking for these special words. Furthermore, key words were found to be misleading. For example, if a problem read: "The boy has 6 bags of marbles with 10 in each bag. How many does he have altogether?", the cue word "altogether" would indicate addition. Potter, Whimbey, and Lockhead (1991) also found that teaching cue words and oversimplified strategies encouraged students to go on "auto pilot" by applying the same strategy to every problem.

Students have a proclivity to focus on answer getting and assignment completion (Winograd & Higgins, 1994) rather than on understanding. Cardelle-Elawar (1990) blame teachers, partially, for this misplaced focus. This researcher claimed that as teachers are pressured to cover material quickly, thinking skills are lost in the shuffle. When students concentrate on "getting the answer," they have no motivation to understand the problem (Winograd & Higgins, 1994).

The last major cause of student failure, covered in this review of literature, is that students often do not allow enough "working time" to problem solutions. Any strategy employed in problem solving requires time for the brain to put together necessary connections. Trahanousky-Orletsky

(1991) did research that showed a primary difference between good and poor story problem solvers was the amount of time they spent on the problem. Their study showed that a good student will persist in working until a mental connection reveals the solution. In contrast, poor students decide quite quickly that they do not know how to solve a problem and give up (Cardella, Elawar, 1990). This quick surrender to failure may be caused by general math anxiety (Verzoni, 1997). Additionally, teachers themselves may be causing this problem when they spend significant class time in rapid fire drill activities (Petter, Whimbey, Lockhead, 1991).

Montague and Applegate (1993) found an interesting aspect to this time on task dimension. They found that how difficult a problem appeared was a more important factor than how difficult the problem actually was. If the problem had the appearance of an easy solution, students would work at it longer in spite of the fact that it actually was difficult. Conversely, an easy problem which at first glance appeared difficult caused students to mentally shut down and give up.

The following is a summarized listing of the probable causes for student failure when solving problems:

1. Student inability to read story problems adequately.
2. Student has low reading ability in general.
3. Student uses improper reading strategies when faced with a story problem.
4. Student does not correctly use problem solving strategies.
5. Student does not seek to understand the mathematical logic of the problem.
6. Student attempts to memorize and apply solutions without problem comprehension.
7. Student does not allow sufficient time for solution.

Problem Evidence

In order to document the extent of student difficulty with problem solving in Site A and Site B schools, the following instruments were used:

1. A student interview (Appendix A) was created to measure students' opinion about math. In general, their attitudes toward story problems, and their initial mental focus when confronted with a story problem.

2. Teacher-made story problem tests at the primary, intermediate, and junior high level were created and given to site students (Appendix B, C, & D). Each test covered the range of mathematical operations used at each grade level and included one and two step problems.

3. The IOWA sub-skills test score on computation and story problem solutions were compared for each student.

4. A teacher survey (Appendix E) was given to Site A and Site B teachers. This survey asked if computation or word problems were more difficult for their classes. The survey also asked teachers to rate probable causes for student failure and measured classroom time devoted to teaching strategies.

Data Analysis Site A, Classroom 1

The student interview was intended to measure both general attitudes towards math and to investigate any anxiety associated with story problem assignments in particular. Site A, Classroom 1 responses were positive regarding attitudes associated with math in general. Nineteen out of twenty students responded "yes" to the question in Item 1, "Do you like math?". In addition, in Item 2, eighteen out of twenty students indicated that they utilized math outside of school. The researcher views this as a positive response towards math concepts as well. The class response to Item 3, "What is the most difficult part of math for you?" was as expected. Students as a whole indicated that word problems were most difficult.

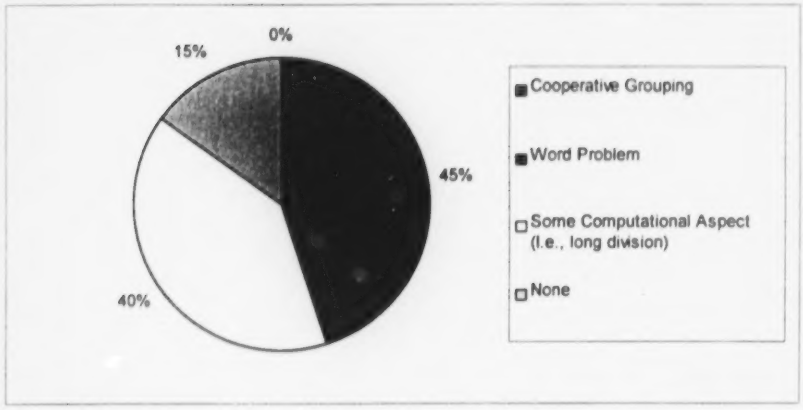

Figure 1. Student response to question, "What is the most difficult part of math for you?" (Site A, Classroom 1).

When Site A, classroom 1, students were asked to write down what area of math was "most difficult" nine out of twenty students wrote down story problems. Following closely behind, in terms of difficulty level, was student computational skills. Eight out of twenty students wrote down a computational skill as "most difficult." Interestingly, three students wrote down that no aspect of math was "most difficult" for them. This data could be interpreted as, these students felt equally at ease with math concepts, or experienced equal difficulty with math concepts. In item four the students were comparing types of math that they disliked and could choose word problems a maximum of five times as their most difficult aspect of math.

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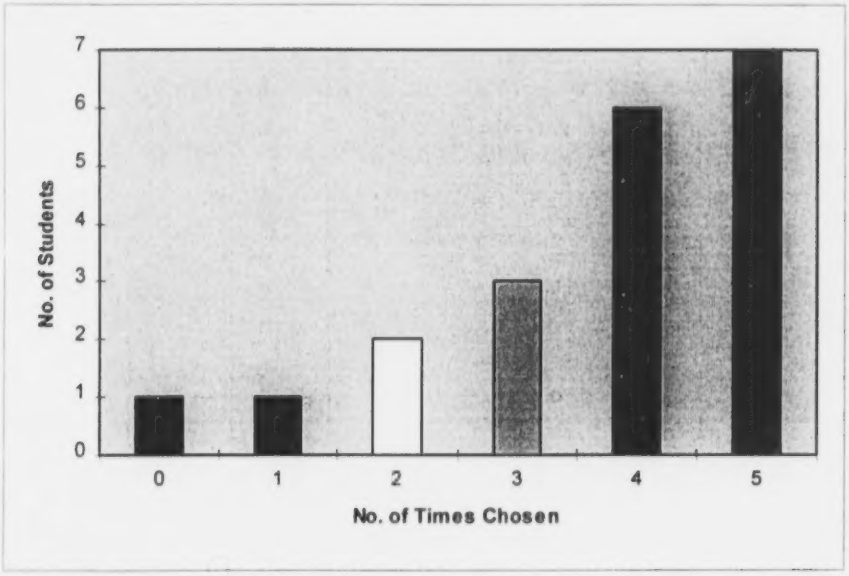

Figure 2. Number of times students chose word problems as "most disliked" (Site A, Classroom 1).

Among 20 students, seven students chose word problems five times, which is the maximum amount of times word problems can be chosen. Six students chose word problems four times. Three students chose word problems three times. Two students chose word problems two times. One student chose word problems only once. Only one student never chose word problems as most difficult. These statistics indicate that all except one student view word problems as a "most difficult" aspect of math when comparing word problems to other areas of math at least some of the time. The last interview question also reflects the majority of students' anxiety towards word problems – with one contradiction.

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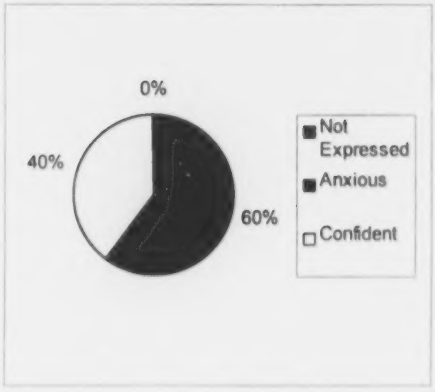

Figure 3. Student's initial reaction to story problem assignment (Site A, Classroom 1).

Despite the majority of students (12 out of 20) expressing anxiety toward word problems, this graph also reflects a great deal of confidence as well. Eight out of 20 students indicated that they were very confident when approaching a word problem. This would seem to directly contradict the data presented in the first four items. However, being confident and being able to understand and answer a word problem are very different issues. Furthermore, students may dislike word problems and still feel confident in their approach to them. Figure 10 offers more clues as to this contradiction.

Figure 4 focuses on analysis of the student responses to item 5. This analysis concentrates on the initial focus of the students when confronting a word problem.

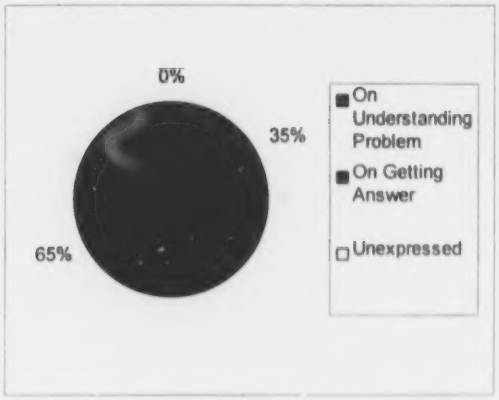

Figure 4. Student's initial focus when given story problem assignment (Site A, Classroom 1).

Thirteen out of 20 students indicated they, are looking for numbers to multiply, add, subtract, multiply, or divide without comprehending the problem. Only seven out of 20 students indicated that understanding the problem should be the first focus. These figures indicate that word problems with extra data or contextual information that requires a comprehension of the contents of the problem would be extremely difficult for the student to solve. Therefore, while the student may be confident in his/her computational skills, he/she may not be applying them correctly. This fact further explains the apparent contradiction in item 5.

In conclusion, the results of the survey point towards a positive attitude towards math with a general dislike for word problems in particular. Furthermore, the students' apparent disdain for the process involved when solving math problems add to the dilemma.

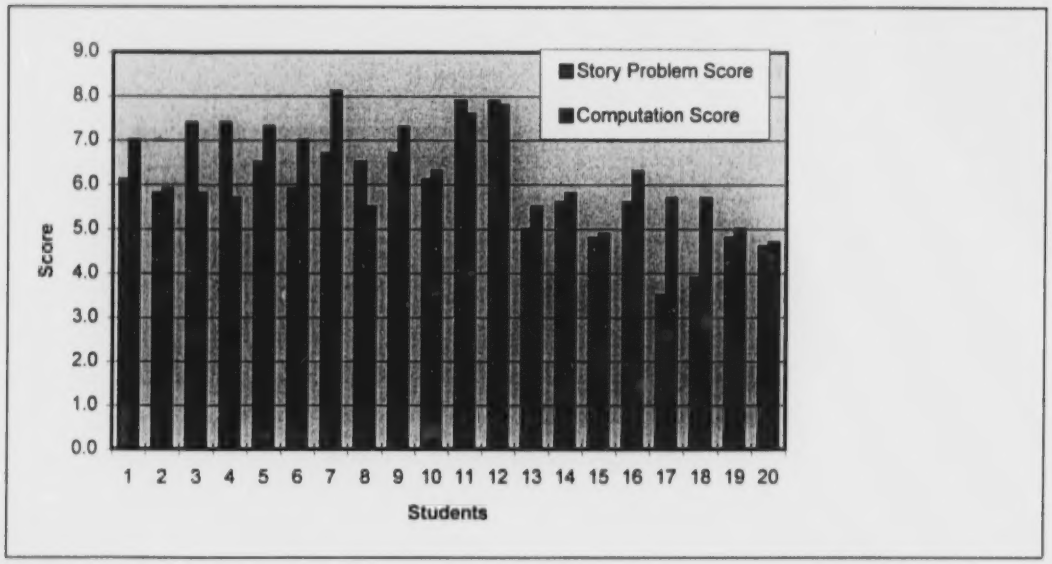

Figure 5. Comparison of student Iowa scores on story problems vs. computation sub-skills tests (Site A, Classroom 1).

This graph suggests that lower performing students do better at computational math than their higher scoring counter parts. The students who scored at grade level or far above had a ratio of 7 to 16 in favor of computation. The students who scored below grade level had a ratio of 4 to 0 in favor of computation. Therefore, the higher scoring students were better at story problems then the lower scoring students. Perhaps this could be related to their reading abilities.

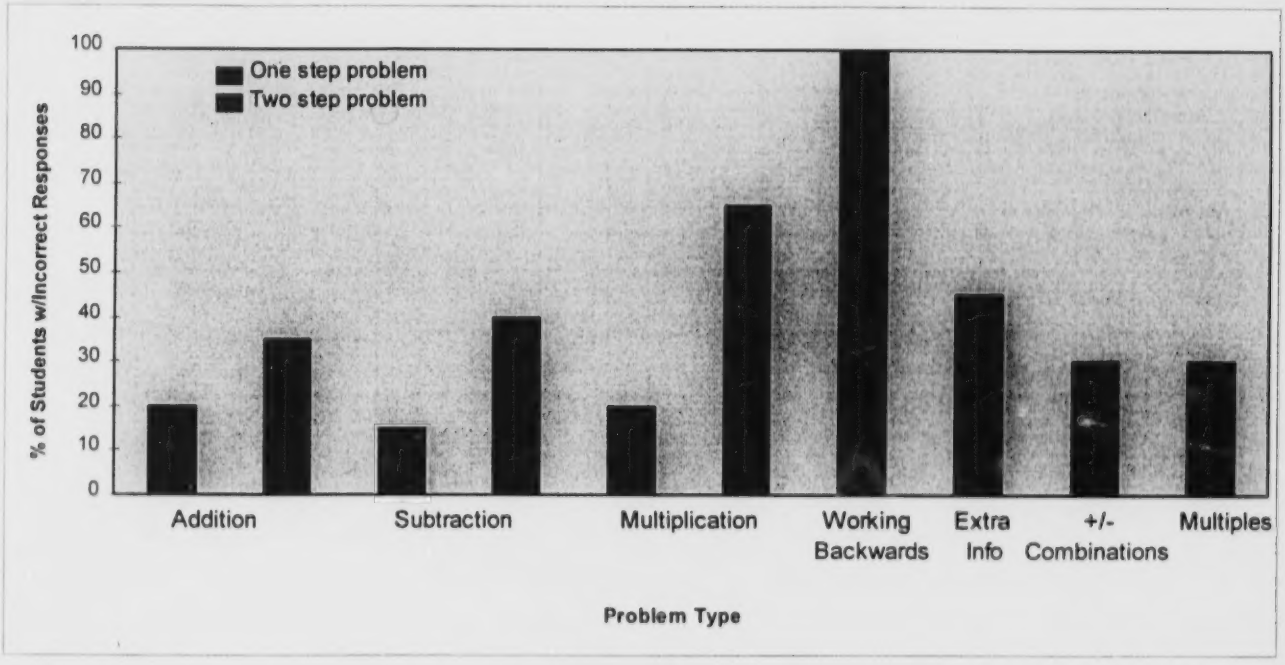

Figure 6. Item Analysis of Teacher-Made Test (Site A, Classroom 1).

The Item analysis of the teacher-made test revealed that by far the most difficult questions were seven and six. Item seven was working backwards which required full comprehension of the question as well as a strategy for approaching this type of problem. Item six was a two step multiplication problem in which the students had to consider not only numerical figures but had to also consider a week was seven days. If the student did not make the transfer of this word into a number they failed the question. Once again it appeared that reading stifled the efforts of the students mathematical success.

Data Analysis - Site A, Classroom 2

The student interview was intended to measure both general attitudes toward math and to investigate any anxiety associated with story problem assignments in particular. Most of the student responses were fairly predictable and seldom surprised the researcher.

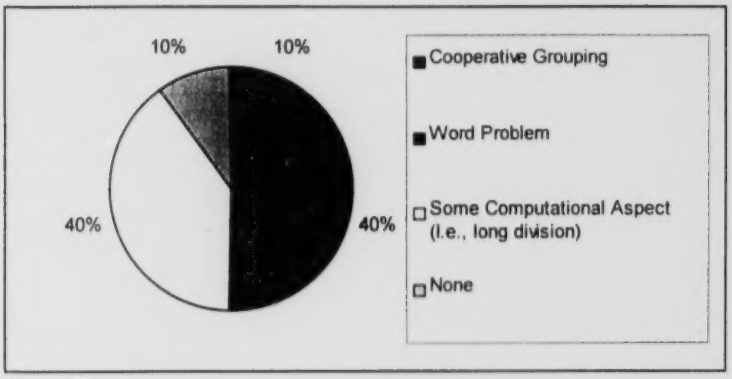

Figure 7. Most difficult aspect of math (Site A, Classroom 2).

In item 3, students were asked, "Which type of math problem is the most difficult?" and responses varied widely. Of the 10 students questioned, only 1 responded "none" while computation and word (story) problems accounted for 80 % of the group. This answer correlates with gathered scores for the students in general. Based on these results, the researcher was now able to focus on specific areas within math word problems that students find difficult. These findings lead the researcher to believe that students need more practice on breaking down word problems into smaller, computational problems, in the hope of eliminating some of the 'fears' students have toward word problems.

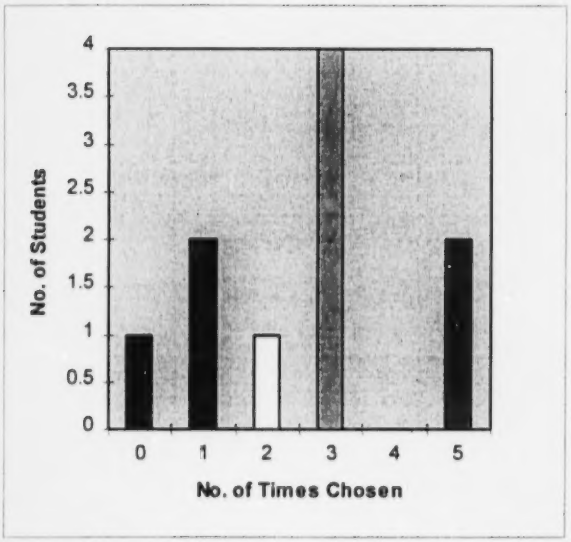

Figure 8. Number of times students chose word problems as "most disliked". (Site A, Classroom 2).

In item 4, students were comparing types of math they disliked and could choose word problems a maximum of five times as their most difficult aspect of math.

Among 10 students, 4 students chose word problems 3 times, 2 students chose word problems one time and another 2 chose them all 5 times. The remaining 2 students were divided between choosing word problems 2 times or not at all. This statistic illustrates that although students often dislike word problems, others (in this case, only one) do not fear them at all. These findings lead the researcher to believe that word problems are disliked more than most any other type of math problem.

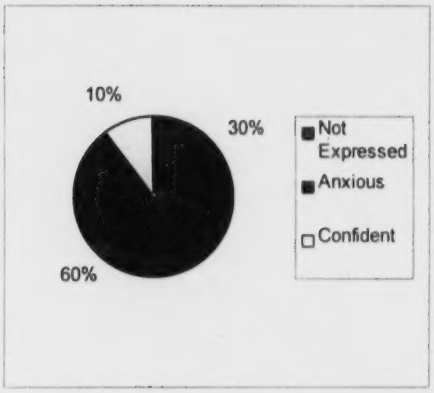

Figure 9. Student's initial reaction to story problem assignment. (Site A, Classroom 2).

Item 5 posed the question, "What do you think of when you first read a word problem?" Results gathered illustrate an extreme level of anxiousness when first faced with math word problems.

Of the 10 students, six answered that they felt "anxious" when given a math word problem to solve. This shows that most students feel some sort of "math anxiety" toward problems which require more than just computation skills. Three other students' responses were not expressed, illustrating a general indifference to these types of problems as compared to any other math problem. Finally, one of the 10 students responded "confident". This low level may explain why so few students score well on these types of problems.

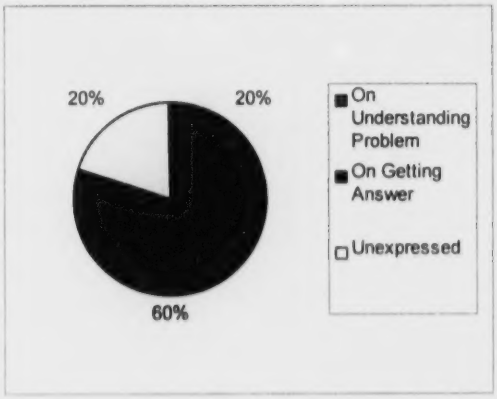

Figure 10. Student's initial focus (when given story problem assignment) (Site A, Classroom 2).

Item 6 posed the question, "What is your initial focus when given a word problem-to understand the answer or to get the correct response?" Obviously, the correct response cannot be achieved unless the problem is understood (other than guessing), so the responses illustrated a lack of understanding regarding word problem strategies.

Of the 10 students, six chose their initial focus as being to attain the correct response. Two of the 10 responded that to understand the problem was their initial focus, while two others were indecisive depending on the problem. Strategies to help change these statistics include graphing the following questions: "What am I being asked?"; "What do I know?"; "What do I need to discover?"; and, "How can I use the information I have to attain the correct response?". In this strategy, the correct response is not even mentioned until three other steps have already been taken.

It appears from the results of this survey that the students suffer anxiety from math word problems. Items four and five support this statement as 40% show word problems as their most disliked aspect of math. In addition, 60% of students initial reaction reflected anxiety. Furthermore, 60% of the students focused simply on getting the answer without understanding the problem. Lastly, Items one and two suggests that students like math. Some of the comments included in their written responses reflected a like for math especially in the students who were good at in that subject.

Another measure of problem evidence was a comparison of IOWA sub-skills test scores. The student scores on the word problem section of the IOWA were graphically compared to the sub-skills for computation.

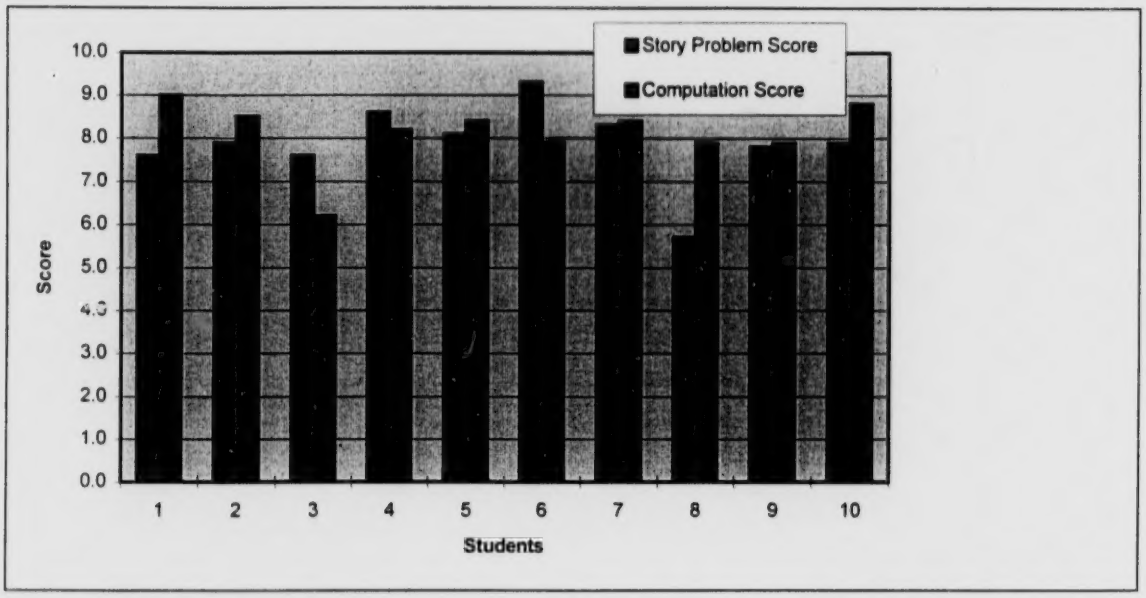

Figure 11. Comparison of Student Iowa Scores on Story Problems vs. Computation (Site A, Classroom 2).

When the researcher averaged the Iowa scores it became evident that the scores do not reflect the gap between computation and word problems. The graph illustrates this to some degree. However, the researcher in order to be more specific averaged the scores in the two sub groups on the Iowa test. The problem solving portion of the test averaged to a 7.9, the computation portion of the test averaged to an 8.2. While this averaging did reveal a slight variance in terms of computation being the more dominant, perhaps this is because the students were able to choose the correct answer from four choices and therefore did not have to generate an answer of their own.

The last student generated set of data aimed at finding problem evidence was the teacher-made story problem test. This was a ten item quiz which covered all the computational skills with which the students were familiar. One and two-step problems were included; 2 "working backwards" problems, one containing extra information and another with insufficient information.

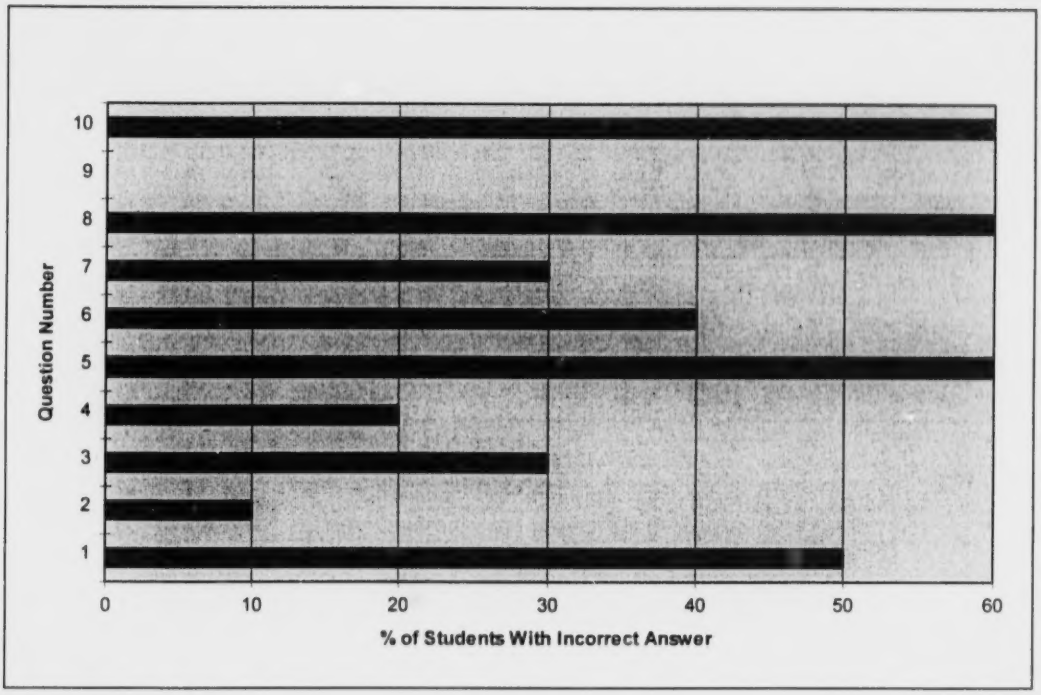

Figure 12. Item Analysis of Teacher-Made Test. (Site A, Classroom 2).

Question Number	Question Type
1	Two step addition
2	Two step subtraction
3	One step multiplication
4	Two step multiplication
5	One step division
6	One step multiplication
7	Working backwards
8	Working backwards
9	Extra information
10	Not enough information

The phrasing of questions played a large role in student success. It appeared that the longer the sentences, the more confusing it was, and thus students did worse. This was the case in questions number five, eight, and ten. When the questions involved information that was relatively new to them (e.g. profits and time) students tended to do poorly also. This was the case in questions

one and six. Lastly, students were successful when the phrasing of the question was clear and the terms were easy to comprehend.

Data Analysis - Site A, Classroom 3

The student study was put together to calculate general views toward math and find any associated anxiety toward story problem assignments. Site A, Classroom 3 had twenty-three out of thirty students answer positively to the question "Do you like math?", and twenty-five of the thirty students answered that they used math outside of school.

The class response to item 3, "What is the most difficult part of math for you?" was not surprising. Most students claimed to have difficulties with the concept of division and did not appear to have confidence with story problems.

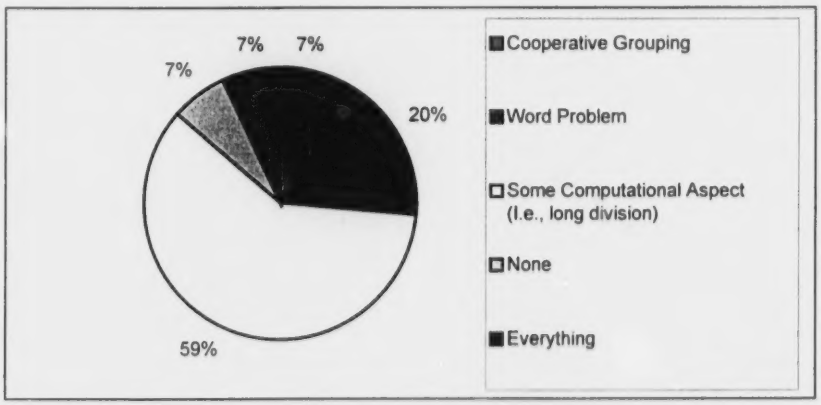

Figure 13. Student responses to the question, "What is the most difficult part of math for you?" (Site A, Classroom 3).

When Site A, Classroom 3 students were asked to write down the area of math "most difficult" to them, only six out of thirty students responded "word problems". Eighteen of the students responded that computation such as division was the "most difficult" part of math.

The responses to item 4 presented the students with five sets of forced choices in which they had to pick a computational skill or word problem as the "most disliked".

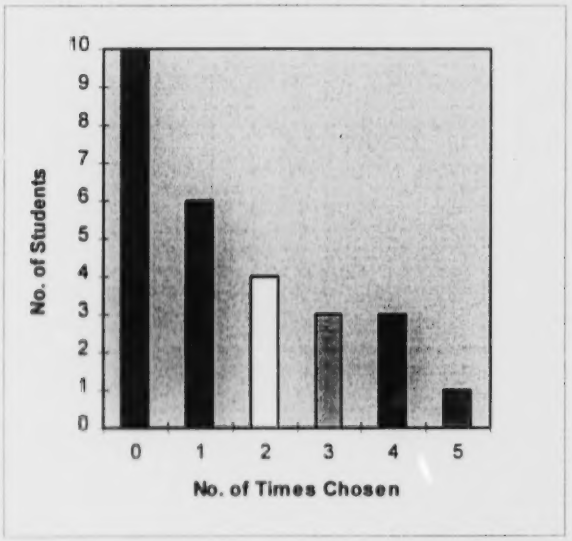

Figure 14. Number of times students chose word problems as "most disliked." (Site A, Classroom 3).

Among the thirty students, ten never chose word problems as the "most disliked", six chose word problems once out of five times, four chose word problems two times out of five, three chose word problems three out of five times, three students chose word problems four out of five times, and one student chose word problems every time.

The student interview, item 4, indicated that a number of students disliked word problems. Item three demonstrated that a greater part of the class found computational problems the most difficult part.

The last question, item 5, "What do you think of when you first read a word problem?" had most students respond either negatively or not at all.

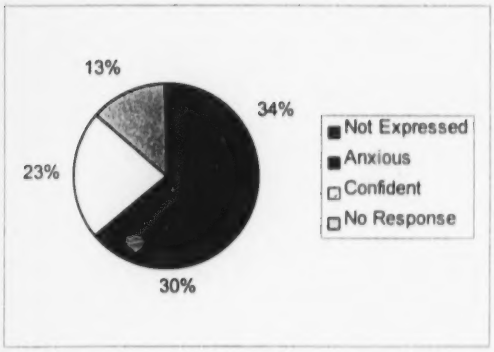

Figure 15. Student's initial reaction to story problem assignments. (Site A, Classroom 3).

Item five was intended to expose any feeling of anxiety held by students when confronted with word problems. More than half the students did not respond to the question or did not express any anxiety toward math word problems. Only nine out of thirty specified apprehension toward comprehension and the difficulty of solving the problems. The comment made by these students were precise and to the point. Some examples are as follows: "That is going to be hard," "how am I going to do it?, what do I do?" and "I can't read fast and it's hard to understand."

Item 5 was used to measure the anxiety levels of students at Site A, classroom three, but also to look at the students' initial focus when confronted with a story problem.

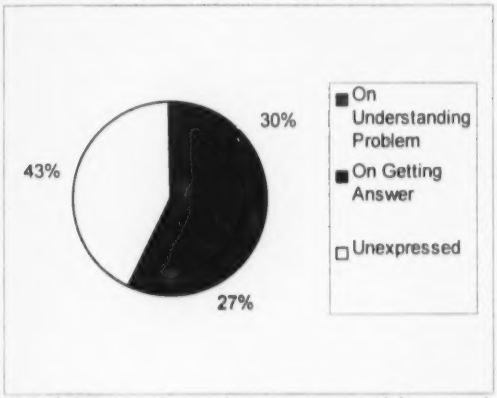

Figure 16. Student's initial focus when given story problem assignment. (Site A, Classroom 3).

When students were asked, "What do you think of when you first read a word problem?", nine out of thirty students answered "understanding the problem" and only eight hinted at using some type of strategy for solving story problems, while the rest (13) did not express any type of concern in solving the problem or providing the answer. The survey did demonstrate that 57% of the students were concerned with the comprehension of the problem in order to get the correct answer.

Another measure of evidence was a comparison of IOWA test scores. The student scores on the word problem section of the IOWA were compared to the sub-skills test for computation on the following graph. The number of students whose scores are shown on the graph is 23 because seven students have no IOWA test score data.

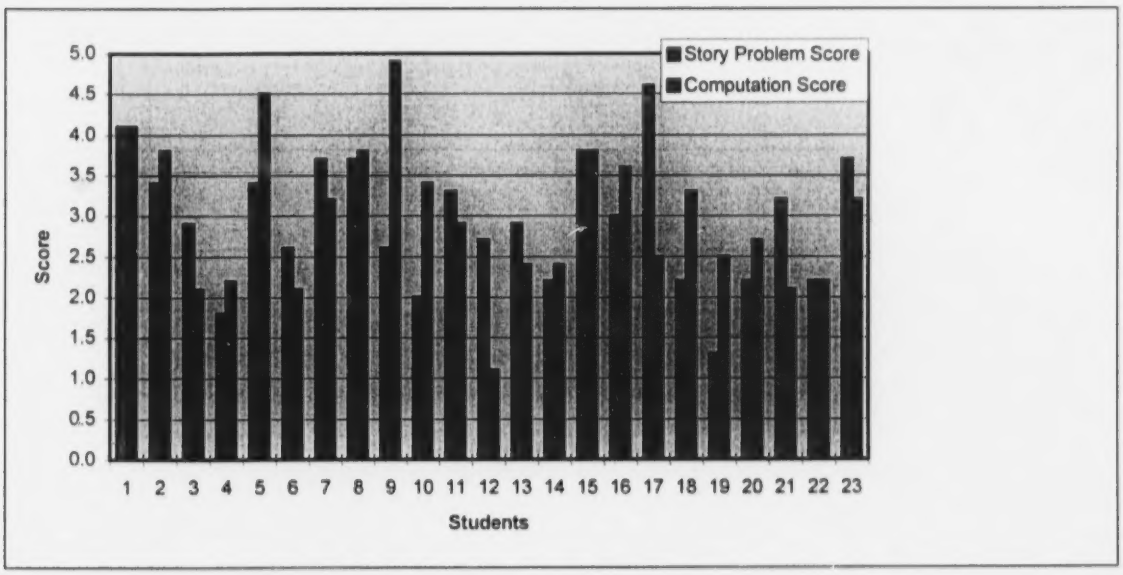

Figure 17. Comparison of Student Iowa Scores on Story Problems vs. Computation Sub-skills Tests. (Site A, Classroom 3).

The students in second grade have approximately the same scores when comparing problem solving to computation. The possible reasons for this could be that when they last took the IOWA, they were second graders and the variable between computation and word problems were fewer than the higher grades due to a limited vocabulary. Given the fact the small variance between problem solving and computation, nine out of twenty-two students performed better in problem solving and three were equal. This variance may be more pronounced in later grades when vocabulary expands. This leads the researcher to believe that in time, as students' vocabularies grow, the variance in student scores may decrease.

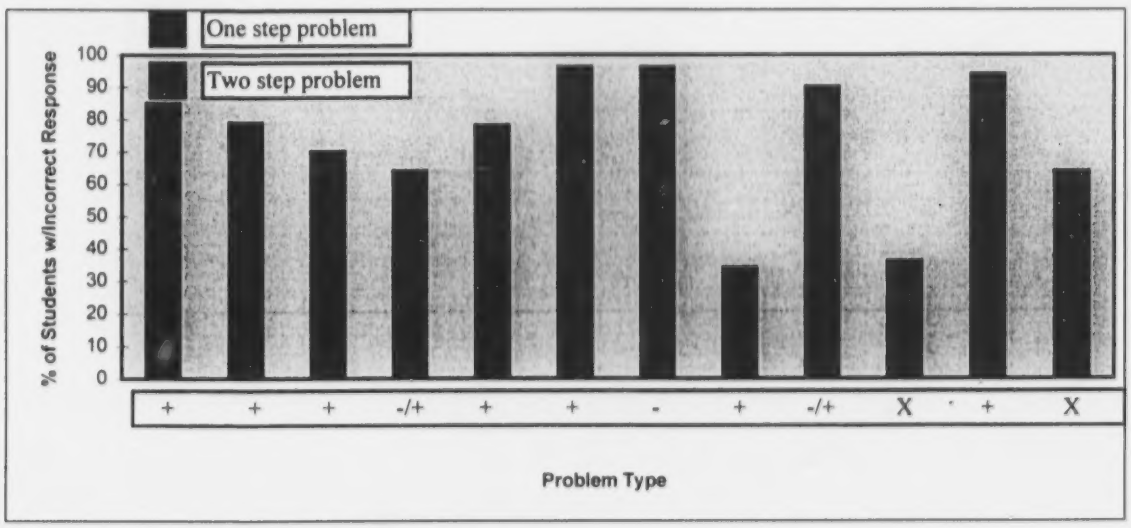

Figure 18. Item Analysis of Teacher-Made Test (Site A, Classroom 3).

This was a quiz that covered all the computational skills which were familiar to the students. This test did not include division because students were not yet exposed to this type of math. The story problems included one and two-step problems involving missing or extra information and working backwards.

The students generally performed poorly on the test involving word problems. The only problem in which they did fairly well on were items eight and ten. Questions eight and ten were the easiest one-step problems which also had the simplest vocabulary utilizing words they have used many times before. The other problems either had more steps or more difficult vocabulary. Reading seems to be the issue here rather than computation.

Data Analysis - Site B, Classroom 1

The purpose of the student interview was to account for the students attitudes towards math and document if any anxiety exists towards story problems. Site B, classroom 1, student responses as a whole were positive towards math. Seventeen out of 20 students responded that they like math. Furthermore, 18 out of 20 students stated that they used math outside of school. These results showed that students feel math is necessary in the "real world."

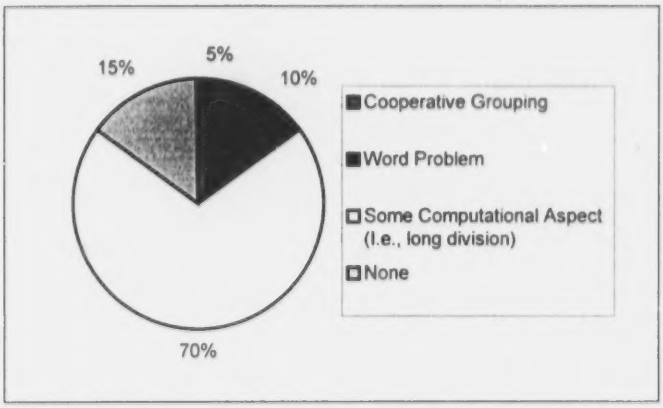

Figure 19. Most difficult aspect (Site B, Classroom 1).

The class results to item 3, " What is the most difficult part of math for you?" came somewhat as a surprise. There were three students who felt nothing was difficult for them. Only

two students chose word problems as the most difficult aspect of math. One student chose cooperative grouping as the most difficult. Lastly, 14 students chose computation as most difficult.

In item 4, students compare types of math they dislike and can choose word problems a maximum of five times as their most difficult aspect of math.

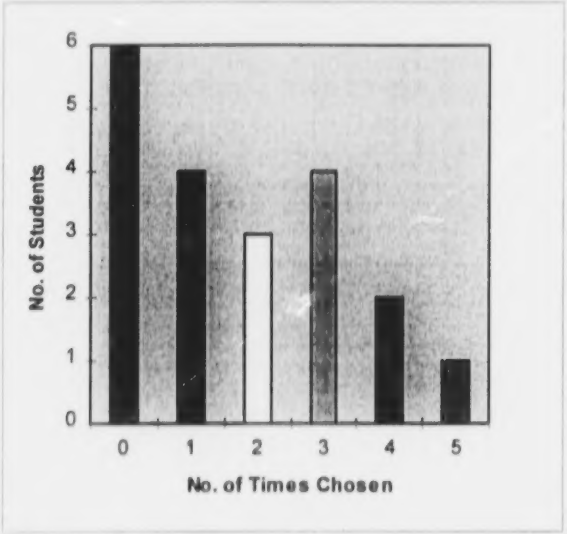

Figure 20. Number of times students chose word problems as "most disliked" (Site B, Classroom 1).

The minimum number of times, zero, was chosen the most (six times) illustrating a high level of confidence in solving these problems. Also, four students chose both one and three times as their number of times selecting word problems as the most disliked. Three students chose word problems three times while two students chose them four times and finally one student chose word problems as the most disliked all five times. These numbers indicate that the students are experiencing a relatively high frustration level when encountering word problems.

Item 5 asked students, "How do you feel when your first given a word problem?" This was posed to find how students confidence levels may be related to scores on these types of problems.

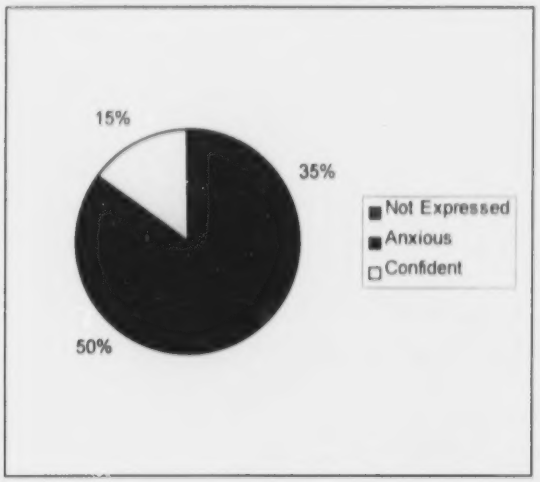

Figure 21. Student's initial reaction to story problem assignment (Site B, Classroom 1).

Half the group responded that they felt anxious when solving word problems, while another 35% were indecisive in terms of how they felt. Only 15% felt confident possibly explaining why students tend to score poorly on these types of problems.

Figure 22 further explains item 5. This portion of the survey was further analyzed with the concentration on students initial focus when given a word problem.

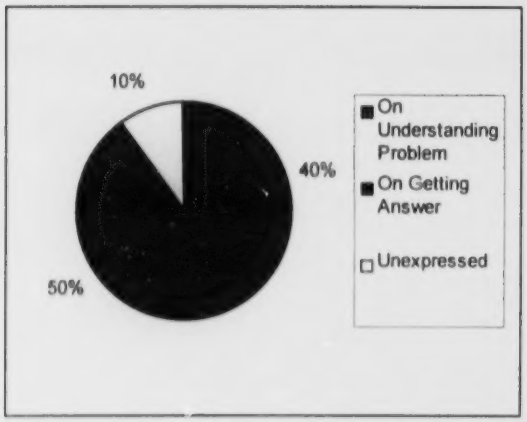

Figure 22. Student's initial focus when given story problem assignment (Site B, Classroom 1).

Surprisingly, only 40% of the class responded that understanding the problem was their initial focus. This is low considering the importance of understanding the problem before hoping to attain the correct response. Half the class responded that the attainment of the correct answer was the first focus and 10% were indecisive. This shows that strategies must be implemented to help students focus more on understanding the problem as opposed to merely arriving at a response.

In conclusion, the results of this survey appear to suggest that the students have considerable anxiety when considering word problems. In item one, 85% of the children's responses were positive when considering math in general. In addition, 90% of the children use math outside of school, as shown in item two. Item three seems to refute the researcher's initial statement because only 10% of the students chose word problems as the most difficult aspect of math. However, 30% of the students in item four responded that word problems were their most dislike aspect of math. Furthermore, 50% of the students expressed anxiety when approaching a word problem. Therefore, even though only 10% of the students chose word problems as the most difficult aspect of math, the remainder of the survey suggests to the contrary. Lastly, in item five, the students initial focus was analyzed. Fifty percent of the students were more concerned with the answer than understanding

the problem. This also could be seen not only as an inappropriate focus but as a possible anxiety type reaction given that they stressed the need for an immediate answer.

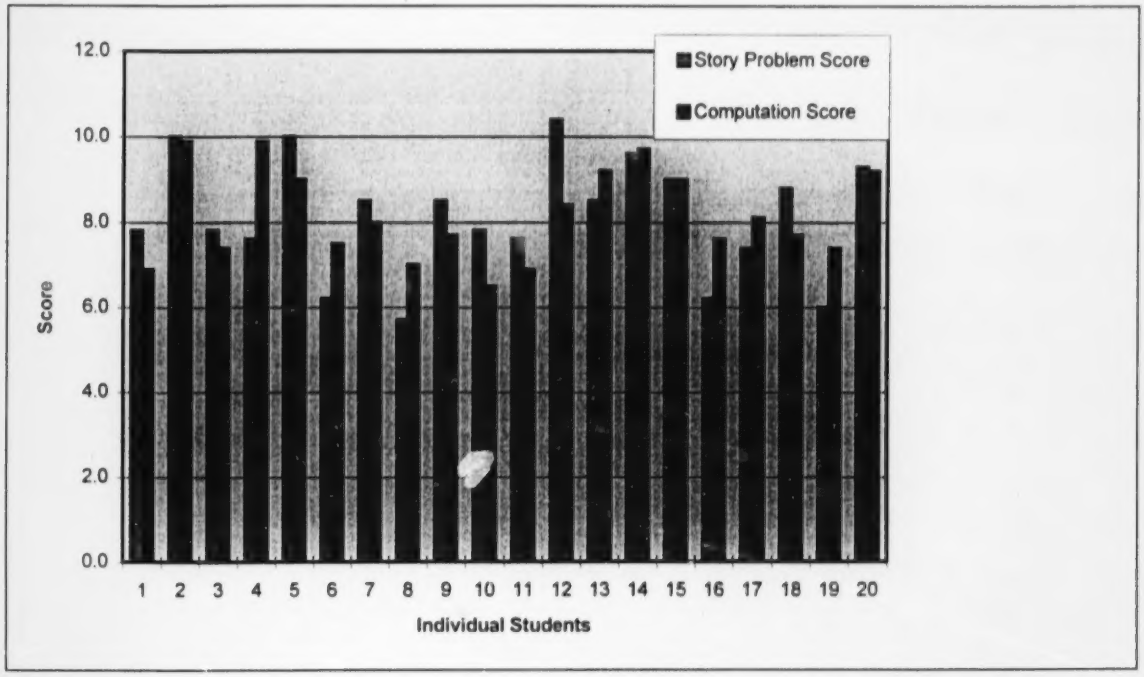

Figure 23. Comparison of student IOWA scores on story problems vs. computation sub-skills test (Site B, Classroom 1).

Given the data represented on this graph, the initial reaction to it could be that the class as a whole performs just as well solving story problems as computation problems. Nine out of 20 students performed better on the story problems section of the Iowa Test. This is nearly 50% of the students performing better on computational skills in comparison to story problem skills. These results may have occurred for the following two reasons. First of all, the students are given the right answer; they merely have to find it on the story problem section of the test. On the teacher-made test they have to complete the problem solving without the benefit of the right answer being present. Secondly, all of the students who performed better in the story problem section of the Iowa Test had

a score that was at grade level or above, suggesting that these students were in general better performing students in the area of math. The students who struggle with math or had a lower than grade level score all scored higher in computation.

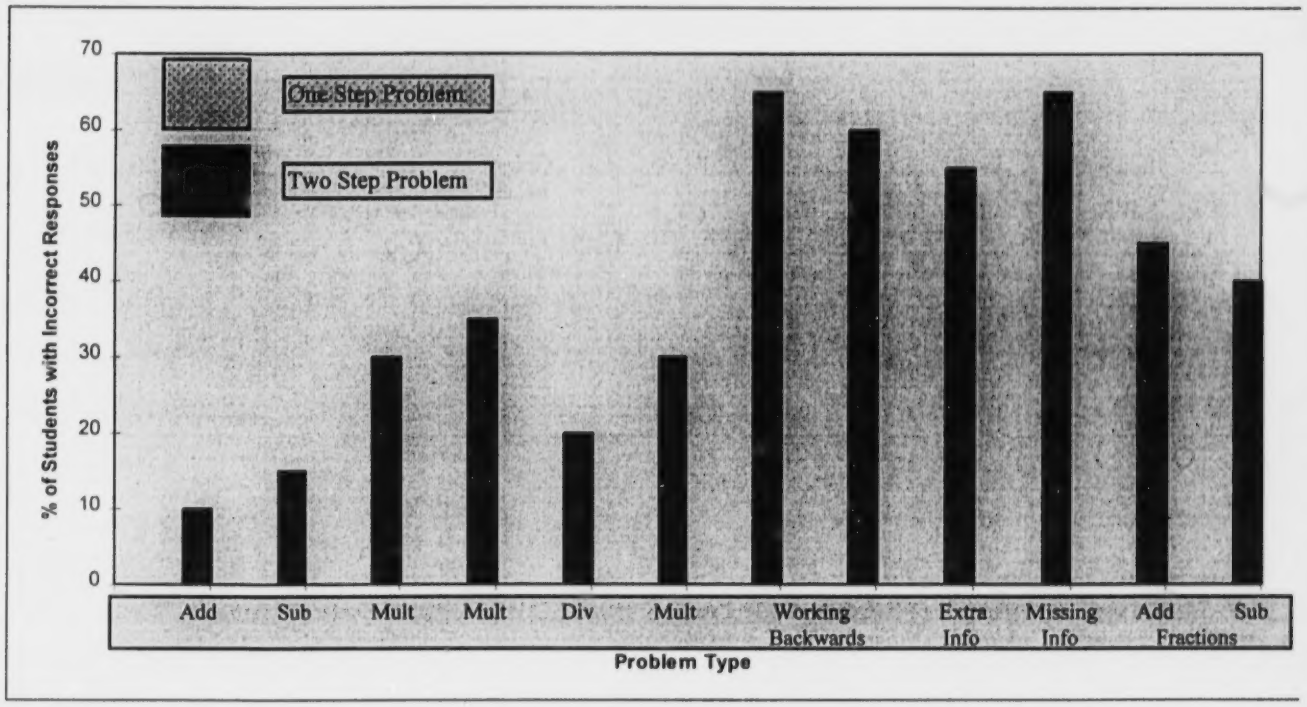

Figure 24. Item Analysis of Teacher-Made Test (Site B, Classroom 1).

The item analysis of the teacher-made test revealed that the most difficult test questions were items ten, seven, and eight. Working backwards and not enough information type questions may have caused more difficulty for the students given that these were the least formulated type questions. To explain, the students had to read and analyze the structure and the purpose of the words included in these type of problems in order to solve them. Therefore, reading skills came into play with a greater emphasis in terms of comprehension in these problems.

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Data Analysis - Site B, Classroom 2

The student interview was intended to measure both general attitudes toward math and to investigate any anxiety associated with story problem assignments in particular. Site B, Classroom 2 student responses were positive to math in general. When answering the question, "Do you like math?", 21 out of 26 students responded, "Yes." In the same positive vein most students felt math was relative to their everyday lives and 24 out of 26 students stated they used math outside of school.

The class response to item 3, "What is the most difficult part of math for you?" was unexpected. First, two students epitomized the group's high self-confidence by stating "nothing" caused them any difficulty. The other survey responses deviated from teacher expectations.

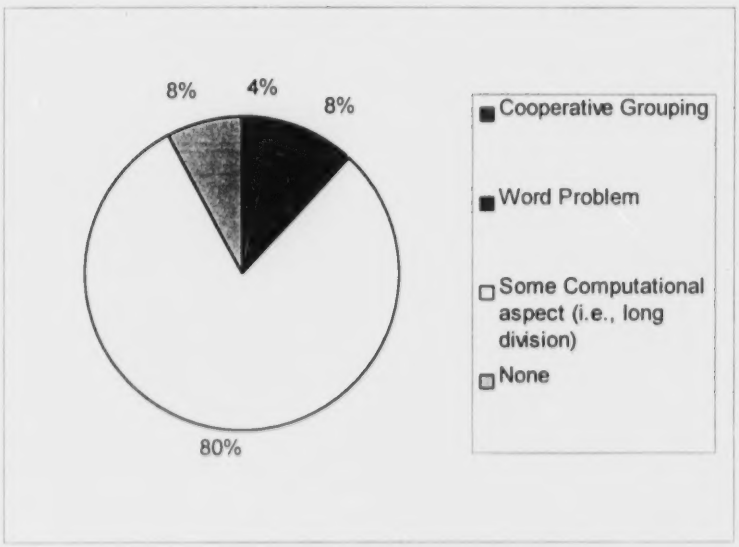

Figure 25. Student response to question, "What is the most difficult part of math for you?" (Site B, Classroom 2).

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When Site B, Classroom 2 students were asked to write down what area of math instruction was "most difficult" only two students out of 26 students wrote down story problems. The majority of students, 21, mentioned areas of computation instead. Long division was the most frequently mentioned skill, being cited by eight students as the "most difficult" part of math.

Responses to Item 4 were also unexpected. This question presented the student with five sets of forced choices in which they had to pick a computational skill or word problems as "most difficult."

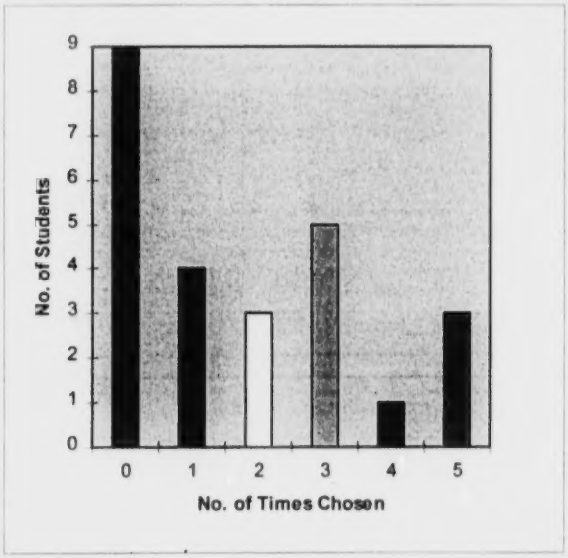


Figure 25. The frequency story problems were chosen as "most difficult" (Site B, Classroom 2).

Among 26 students, nine never chose word problems as most "disliked"; four chose word problems once out of five times as most "disliked", three students chose word problems two times out of five; five students chose word problems three out of five times; one student chose word problems four out of five times; and three students chose word problems every time.

At first glance, items three and four of the interview appear to indicate that students in this class did not have difficulty with word problems since the majority of students chose computational problems over story problems most of the time.

However, the last interview question, item 5, "What do you think of when you first read a word problem?," revealed a contradiction.

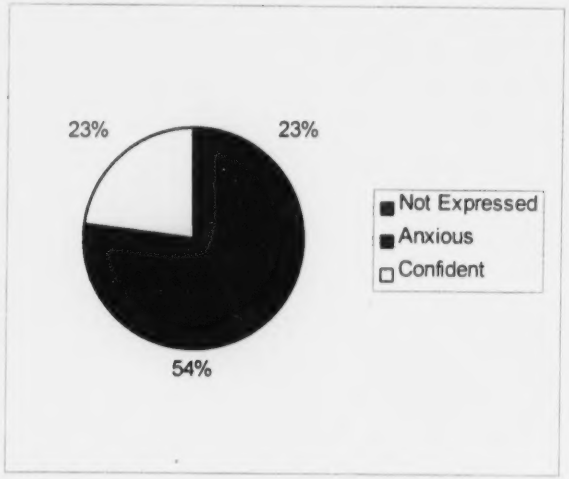


Figure 26. Categorization of student attitude revealed in Item 5 response (Site B, Classroom 2).

Item 5 was intended to reveal any feelings of anxiety held by students when confronted with word problem assignments. In spite of the lack of indictment of word problems as most disliked or difficult from Items 3 and 4, more than half (14 out of 26) expressed anxiety about doing a story problem. Student comments were not difficult to rate as anxious since most negative responses were fairly to the point. Many students expressed worry over failure. For example, typical comments were, "like I'm going to get it wrong," "I start to get nervous," and "it gets confusing." Often anxiety was expressed over problem difficulty. For example, "When I first read a word problem I thought it was hard," is a typical comment that was rated as "anxious".

An examination of the students' previous math experience may illuminate the contradiction of responses to items 3 and 4 with anxiety levels expressed by responses to item 5. The majority of these pupils are fourth graders, who have not yet been instructed in many of the mathematical choices found in item 4. Unfamiliarity with choices such as decimals and division may have prompted their choice as "disliked" over story problems since an unknown skill may seem most threatening. A second factor that may cloud the picture of students' attitudes is that most third grade math curricula do not allocate significant time on task for word problem activities. When asked what *caused* most difficulty, students may have remembered what they spent the most time doing in math - mainly computation. If only a minimum of time was spent working story problems, memories of difficulties would of course center around computation.

The main purpose of item 5 was to measure anxiety toward story problem tasks. However when analyzing the responses, a second factor was revealed in many of the student answers: The initial focus of a student as he/she faces the assignment.

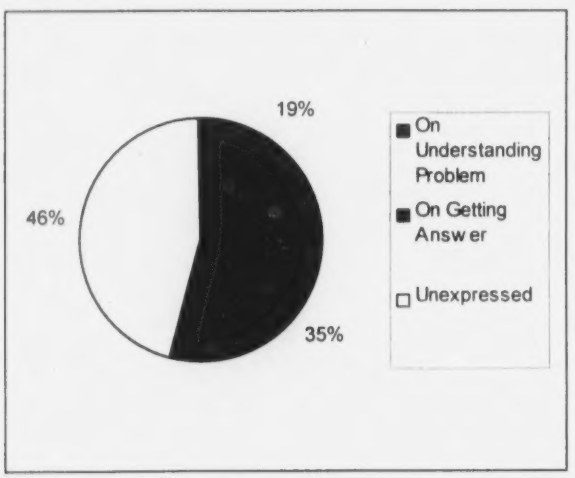
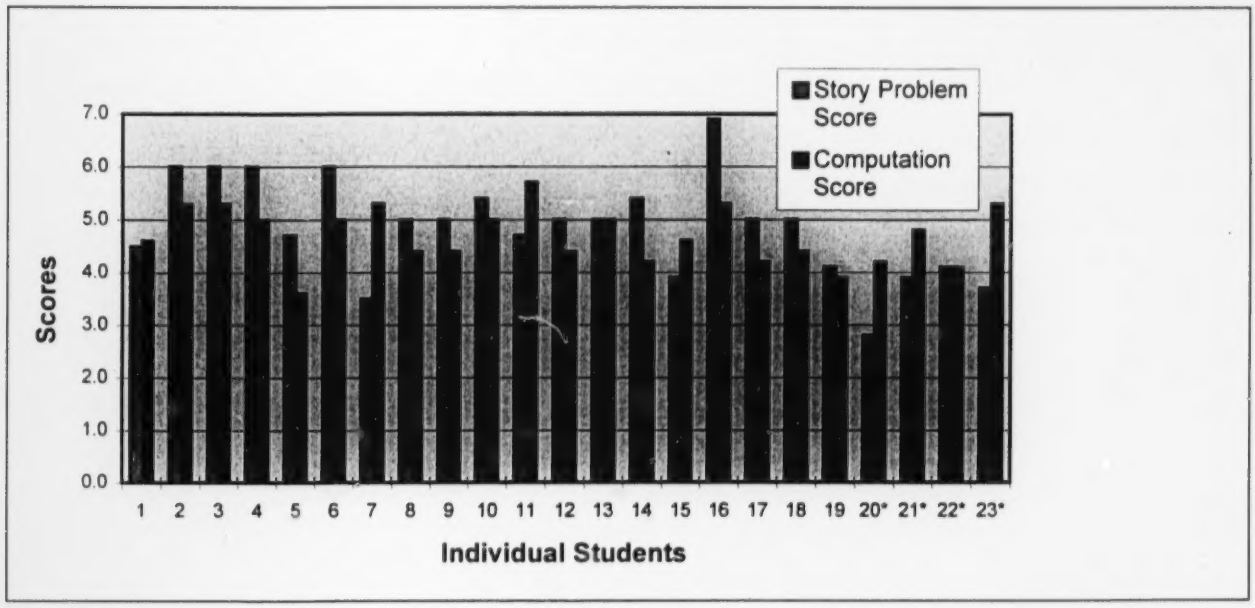


Figure 27. Categorization of student initial focus revealed in Item 5 response (Site B, Classroom 2).

When asked, "What do you think of when you first read a word problem?", nine out of twenty six student responses focused on answer getting over problem comprehension or strategy use. For example, "I try to think of the answer"; or "I just thought of adding or subtracting." Only a few (5) made even rudimentary hints of strategy use, such as "read it over." The fact that only five of 26 students revealed any purpose linked to strategy use or problem understanding correlates to the research literature (Winograd & Higgins, 1994) which found that students do not consider understanding the problem; they usually focus on "answer getting."

In conclusion, the survey did indicate evidence of story problem anxiety and dearth of strategy use in Site B, Classroom 2. Fifty three percent directly expressed feelings of worry and doubt about being successful at problem completion. Furthermore, only five out of twenty six students indicated a goal of understanding the problem or of strategy implementation when confronted with story problem assignments.

Another measure of problem evidence was a comparison of IOWA sub-skills test scores. The student scores on the word problem section of the IOWA was graphically compared to the sub-skills test for computation.



* - denotes 5th grade students

Figure 28. Comparison of standardized test scores of individual students on computational and story problem sub-skills tests (Site B, Classroom 2).

The uniqueness of this class is revealed by the generally high IOWA scores. First, a high percentage of the class (74%) had story problem test scores equal to or higher than their computational test scores. Second, 74% of the class had both sub-skill test scores at or above grade level (3.8 for the fourth grade students, 4.8 for the fifth grade students). Finally, when comparing the fourth grade and fifth grade scores, one sees that 79% of the fourth grade had story problem scores equal or greater than their computational score whereas none of the fifth graders had a story problem score higher than their respective computational score. One fifth grader had equal scores on both tests. Additionally, no fifth grader had both sub-skill test scores at or above grade level.

The last student generated set of data aimed at finding problem evidence was the teacher-made story problem test. This was a ten item quiz which covered all the computational skills with

which the students were familiar. (Division problems were not included for Site B, Classroom 2 because students had not been instructed in that algorithm yet.) One and two step problems were included; a "working backwards" problem was given; a problem which contained extra, misleading information was included; and finally a story problem which involved multiples (i.e., twice as many) was given.

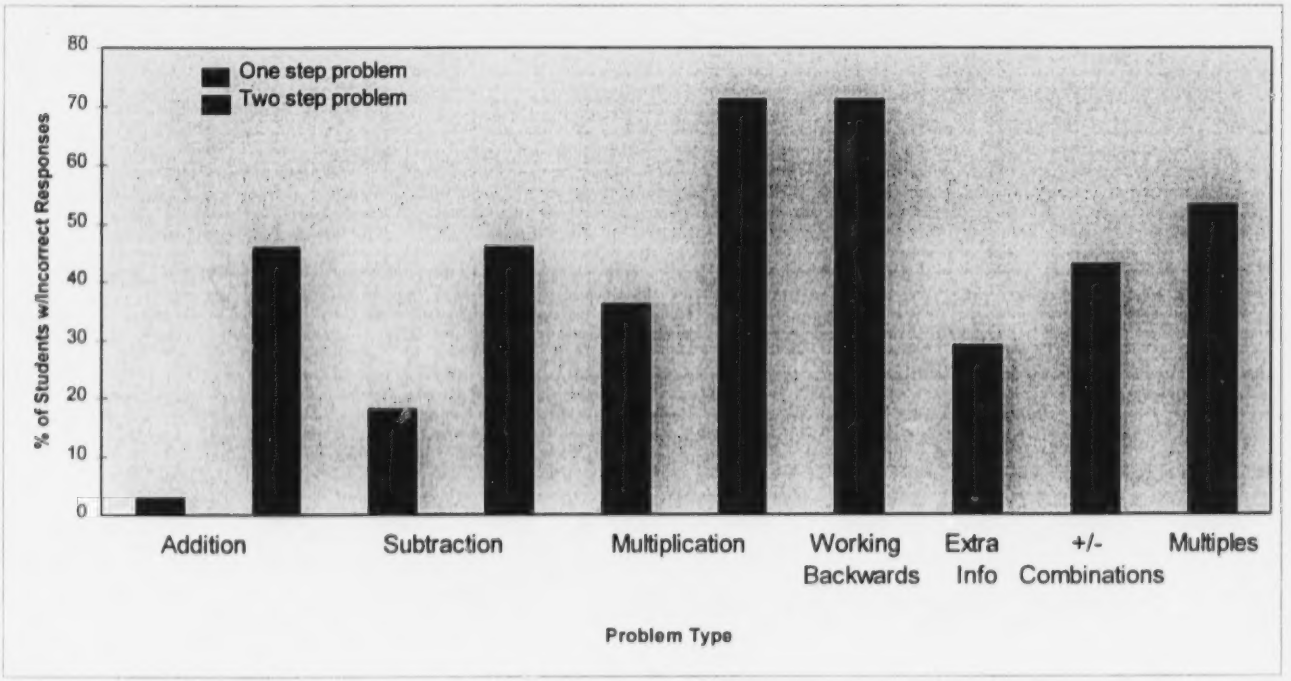


Figure 29. Analysis of % of students responding incorrectly to test items on teacher-made test (Site B, Classroom 2).

The X axis indicates each type of problem represented in the test. The Y axis indicates what percent of the class failed to correctly solve the problem. All answers were marked "correct" if an acceptable problem-solving procedure was used; math computational errors were not marked incorrect.

The test indicates that these students do need further instruction in word problem strategies. The only problems solved by most students were the one step addition and one step subtraction problems. For most of the other problems the failure rate exceeded one third of the class. The failure rate for two step problems was even higher, ranging from 43% for a combination addition and subtraction problem to 71% for the two step multiplication problem.

Although this test did indicate that an intervention was warranted, the scores of this class on the test were much higher than anticipated. Therefore, the same test was administered to another fourth grade class at the same site to see if the scores were anomalous due to the students' high math abilities or to the test questions being too easy. The significantly large difference between the two classes mean, median, and mode on the table below supports the test's validity.

Table 1

Comparison of Teacher-made Test Data for Site B, Classroom 2 with Another Class

Score Analysis	Classroom 2	Another 4th Grade
Median	60	20
Mode	70	20
Mean	50	36

Teacher Survey

A teacher survey (Appendix E) was distributed to classroom teachers at both sites to determine how they interpreted student attitudes toward math in general. Teachers noted their feelings toward teaching computational formulas and strategies as compared to story problem

strategies. Finally, teachers noted the amount of time they devote to teaching math word problem strategies on a regular basis.

According to the teacher survey, all teachers responded that students, in general, perform better on math computational problems than on story problems. Similarly, teachers responded that students tend to perform better on the computational portions of the IOWA test of basis skills, as opposed to the story problem sections.

In terms of teaching lessons, teachers overwhelmingly agreed that story problem strategies were the most difficult to explain to students. Nearly 75% of teachers chose "story problems" when asked, "What types of math lessons are most difficult to teach?" The other quarter chose either "computation problems" or "other".

Five questions were posed to teachers asking them to focus on specific areas of difficulty (in terms of word problems) for many students. According to the results, 80% of teachers agreed that poor reading skills contribute to student difficulty, and 88% agreed that students lack strategic abilities concerning these types of problems. Sixty-eight percent of teachers believe students lack self-confidence to solve word problems, while 72% responded that students have a "let's just get an answer" attitude. Fifty six percent of the teachers noted that students spend an insufficient amount of time on task when attempting to solve word problems.

Finally, teachers were asked how often they taught word problem strategies on a regular basis. Sixty percent responded that they taught word problem strategies "on a weekly basis", while 20% answered, "whenever the text indicates a need". Twelve percent of teachers responded that strategies were taught "on a daily basis", and the other 8% "on a bi-weekly basis".

CHAPTER 3

SOLUTIONS

Literature Review

In Chapter Two, the writers explored the research literature that revealed various causes for student difficulty with problem solving. These underlying causes included students' lack of reading skills and faulty teaching strategies. However, most studies referenced pointed out that the primary cause of student failure was a lack of correct application and processing of thinking skills (Cassidy, 1991; Potter, Whimbey, & Lochhead, 1991; Miller, 1996; Winograd & Higgins, 1994; Parmar & Cawley, 1994).

A major force in educational views today is the philosophy and teaching of the mathematician George Polya (1887 - 1985). He became a huge influence in the field of math education because his techniques addressed the issue of student thinking which is so pertinent to math achievement. Polya was one of the first to teach problem solvers to use metacognitive strategies. First, he taught that guessing was not a random activity but a valuable problem solving mental tool (Sinicrope, 1995). Guessing solutions and trying out various methods of attack was part of a purposeful plan. Polya also set up a heuristic set of rules to regulate the thinking process of "good guessing," many of which can be found in student textbooks today. For example, one strategy is to simplify the problem in an attempt to clarify the math question.

The use of visual representation to improve understanding is integral to this strategy. "Good guessing" was defined as using inductive reasoning; if the student could discover a solution by putting the facts together himself, real learning took place. Second, a very important part of the discovery process was the seeking of analogies and patterns (Leinhardt, Gaea, Schwarz, Baruch, 1997). A look at most textbooks today would reveal that they devote significant time to pattern finding. Finally, Polya taught that checking and testing the solution was a thinking skill and an integral component of his heuristic method (Sinicrope, 1995).

Our investigation of educational research revealed that many aspects of Polya's philosophy are the backbone of current solutions to student difficulties with math problems. For example, many educators today emphasize use of metacognitive skills and recommend methods of developing these skills. Howell & Barnhart (1992) found students could learn to think about math concepts successfully by using the following three-step process. First, expose the student to concrete, hands-on experiences. Next, move the student into representational actions such as using drawings and symbols. Third, teach math concepts through the abstract construct of equations.

Many students today have been taught using the last step only. This educational mistake is addressed repeatedly in the literature. Mastromatteo (1994) goes even one step further into emphasizing the use of concrete manipulatives by suggesting students physically act out the problem. Assignments in which children physically explore their environment through measurement (Cradick-Oppedal, 1995) or role-play real life experiences such as making travel arrangements (Emenaker, 1997), are examples of how to add the dimension of physically acting out a story problem for a math lesson.

Many of the strategies used to teach math thinking skills are similar to Polya's heuristic model. For example, Howell & Barnhart (1992) recommends students use an attack strategy with the following five steps: First, recognize the problem. Second, organize the data. Third, devise a plan. Fourth, do the computation. Fifth, check the answer. Montague and Applegate (1993) address the students' metacognitive strategies when problem solving by using a similar but more cognitively detailed strategy. Their method includes these steps: First, read the problem. Second, paraphrase what was read. Third, visualize the components of the problem. Fourth, hypothesize solutions. Fifth, estimate the answer. Sixth, do the computation. Last, check the answer.

In addition to developing metacognitive awareness, other thinking skills need to be addressed. For example, students need to become proficient in the thinking skill of analysis; they especially need to analyze the data given in a word problem. Potter, Whimbey, and Lockhead (1991) found that students were likely to lump all the numbers together in a single computation instead of doing a step-by-step analysis of the information. One teaching technique they recommended to overcome this habit was to create a sequence of problems that all used the same numbers but required different mathematical operations.

Another method of helping students analyze data is to provide them with a structured format, such as a numerical table (Mastromatteo, 1994) or a graphic organizer of information (Miller, 1996). Trahanovsky and Orletsky (1991) created an encompassing problem data grid. It forces students to do a detailed analysis of the problem by filling in nine different boxes for data including such information as key words, "noise" (non-pertinent information), conditions set up in the problem, and hidden numbers. The authors stated that use of such a tool had benefits beyond that of teaching analysis skills. Its use encouraged multiple readings of the problem and

prolonged mental attention to the word problem's various components. This effort allowed the student's brain time enough to make connections and discoveries.

This goal of discovery and understanding is often elusive to students. Many students fail to see problem solving as a learning process; rather it is an "answer getting" process. Some researchers feel that bringing the problem out of the abstract textbook and into the students' real world will promote growth of understanding. Mastromatteo (1994) found it very motivating to incorporate student real life facts into the word problems; this technique encouraged students to see these problems as something they could understand (Smith, 1995). Teachers should remove the story problem from its rather divorced from reality text-based format and put it into the students' real life experience.

In addition to using student-based facts in word problems, this goal can be accomplished by incorporating math lessons with language arts classes. Winograd and Higgins (1994) recommend applying whole language concepts to problem solving instruction. For example, math problems become authentic when students write their own stories; they take ownership of the story's problem when they are its author. Winograd and Higgins (1994) suggest that classroom teachers set up an "author's chair" lesson structure similar to what has been used in language arts writing instruction. Children could write out math problem vignettes based on their everyday life and pose these problems to their classmates. The researchers felt such an activity would increase math reasoning and application skills. Other research bears this opinion out. Potter, Whimbey and Lockhead (1991) found that writing their own story lines increased students' ability and motivation. One side effect which may increase the motivation factor was that student written problems were both more relevant and entertaining than text problems.

Another method suggested to increase problem comprehension through written expression was to present the class with an equation and have the students write the story behind it (Howell & Barnhart, 1992). For example, the teacher would put 225 times nine on the board and each child would have to create a story problem that would fit that equation. Cassidy (1991) also suggests students alternate between having to solve a problem with an equation and creating their own story problem to match a given equation.

There are many benefits to incorporating story problem instruction with language arts and other subjects. For example, the thinking skill of predicting is commonly taught during reading. However, the teacher may beneficially use prediction as a thinking skill applied to mathematical comprehension by giving students word problems without the final question (Cassidy, 1991). When students guess or predict what the problem will ask they are transferring thinking skills usually used in reading to the domain of math and are increasing their expertise in both subjects. Other researchers recommend connecting story problems to current reading, science, and social studies instruction because of the benefit to poor readers (Miller, 1996). Since many of the vocabulary words and prior knowledge concepts have already been explained when presented in another subject, a low-level reader has a better chance of being able to comprehend the math problem (Parmar & Crawley, 1994). In fact, when reading ability is a serious consideration, Parmar and Crawley (1974) suggest treating the story problem the same as a reading lesson. The teacher needs to activate prior knowledge, teach vocabulary, and attend to reading comprehension needs. Comprehension strategies were further applied by McIntosh (1997) who found one successful method of teaching students how to do math word problems was to apply the three-level reading comprehension strategy. Students were taught to analyze problems with literal, interpretive, and applied comprehension techniques.

Incorporating language arts with math lessons needs to be done in the area of oral expression as well as written expression. Students are often required to verbalize while they complete an algorithm; they should be verbalizing their thoughts at the conceptualization stage (Potter, Whimbey, Lockhead, 1991). Cardelle-Elawar (1990) pointed out two desired benefits from teaching verbalization. First, when teachers model this process they are teaching students how to critically review their own thinking process. This helps impart the metacognitive learning necessary for successful problem solving. Secondly, teachers can catch errors during the process of problem solving. The teacher's feedback during the thought process is much more likely to promote learning than is correcting the student's final answer. Faulty reasoning can be corrected on the spot. While some teachers feel that having students think aloud will lower the quality of the student's concentration, Montague and Applegate (1993) found no evidence to support that fear. Furthermore, verbalization was found to be one of the most accurate methods of assessment of student understanding (Howell & Barnhart, 1992). Once students can verbalize how to solve a problem, the teacher is assured they have mastered that solution.

Besides the ability to verbalize solutions, teachers need to help students visualize problems. Mullis (1992) reports that illustrating the problem promotes clear thought processing. From Polya (Leinhart et.al., 1997) to more modern math educators, illustration has been promoted as an excellent method of assisting the student in his conceptualization of a problem (Potter et al., 1991). Besides students manually drawing out problems, research has also indicated that teachers should help students increase their mental spatial sense (Reynolds, 1997). For example, using visual metaphors, such as "chopping" for division is a good way to develop mental visualization. The ability to visualize mathematical concepts is so important that many educators include some form of visualization in their heuristic methods. For example, Montague

and Applegate (1998) use visualization as the third step in the problem solving strategy described earlier in this paper. Miller (1996) also concluded that drawing out the problem is integral to both the solution and the checking of the answer.

Verbalization and illustrating are not only recommended by research. Current state testing procedures as well as national assessments (Mullis, 1992) are demanding more than computational skills. Students are being required to explain and illustrate their problem solutions.

A further suggestion directed toward teachers who wish to improve their math curriculum is to incorporate the use of a grid of story problem classifications (Carey, Carpenter, & Fennema, 1993). This will insure that the students will be exposed to a diverse blend of story types. Parmar and Crawley (1994) created an example of a classification grid of problems. In their chart, problems are grouped according to operation, number of steps necessary for solution, and use of misleading details. Using a systematic approach such as this both insures exposure to a variety of problem types and may be helpful when diagnosing student weakness. For example, a student may have trouble discerning misleading data in a problem. Using the above grid would make that weakness easy to diagnose.

One cause of student difficulty noted in Chapter 2 was anxiety when working with word problems. The most successful solution to this nemesis is the use of cooperative learning structures when having problem solving lessons. Mastromatteo (1994) found that working in pairs and small groups increased student achievement. Students report that they prefer cooperative group class structures for this kind of assignment and are more motivated to work when they are used (Potter et al., 1991). One explanation of this group success is that students with low confidence will give up quickly when presented with a challenge (Cardelle-Elawar,

1990). When working in a group, that same student may be motivated to continue to work on a problem long enough to find the solution.

In looking for solutions, the primary encouragement for the teacher is the documented evidence that regular and direct instruction does improve student achievement in problem solving. Potter et al. (1991), found that just two lessons a week for three months created real benefits. When students get used to the daily math "circle" (Winograd & Higgins, 1994) mathematical thinking becomes more natural for them. As in all other areas of instruction, time on task is an important consideration. Following the recommendation of putting story problem instruction into the daily routine has been shown to increase student achievement (Miller, 1996).

Project Objectives and Processes

As a result of increased instructional emphasis on math word problem-solving processes during the period from September, 1998 to March, 1999, elementary level students from the targeted classes will increase their expertise in solving math word problems, as measured by teacher-constructed tests, weekly work, and observational checklists.

As a result of increased instructional emphasis on math word problem-solving processes during the period from September, 1998 to March, 1999, students from the targeted classes will decrease their anxiety associated with the processes involved in solving math word problems, as evidenced by peer interviews and reflective math journals.

In order to accomplish the project objectives, the following processes will be implemented:

- Problem of the Day - students are challenged to use learned strategies in solving daily math word problems.
- Cooperative Groups - students assume roles while working together to solve math word

problems.

- Pair/Share - students use manipulatives to solve "real life" word problems with a partner.
- Draw Pictures - students sketch diagrams and draw pictures to represent information in word problems in order to find solutions.
- Math Journals - students use journals as enrichment for math word problems.
- Classification of Word Problems - students are taught strategies according to the four basic operations of math in order to alleviate math anxiety. (Appendix F)
- Analytical Worksheets - students document, in order, the steps required to solve math word problems. This process helps students to work in a step-by-step. logical manner. (Appendix G)

Action Plan

In order to address the issues involved with problem solving skills, this action plan will incorporate seven different strategies in a simultaneous manner. Due to this format, the action plan will be presented in narrative form. Intervention strategies will be introduced on Sept. 7th and conclude on Mar. 5th. The following models will be used.

Classification of Word Problems:

Word problems will be divided into the four basic operations of addition, subtraction, multiplication, and division. Each operation will be further categorized by recognizable characteristics (Vandewalle, 1997). For example, under the operation of multiplication, students will have rate, combination of choice, row by column, and repeated addition problems. Different categories will be taught as separate units. Students will be taught vocabulary, recognizable traits, steps to solve, and real-world associations involved with each category.

Problem of the Day:

The problem of the day will be implemented on a daily, basis and will correlate with the category of word problems that are currently being addressed. The teacher will create situational word problems that directly relate to the experiences of the students. The teacher will focus on one strategy at a time. Students will have one full school day to complete each problem.

Cooperative Grouping:

Cooperative groups will be utilized as a catalyst for the other strategies contained herein. Within cooperative groups, students will be able to share experiential knowledge so that word problems can be related to the real world. In forming cooperative groups, students utilize interpersonal skills to transfer information among members. In addition, students with a poor sense of visual imagery may gain a better understanding of given problems following peer discussions. This type of grouping also ensures that all group members are actively involved in the solution process. Students may be assigned roles which focus on their particular strengths or weaknesses.

Pair/Share:

Students will be paired in a 'buddy system" in which average to superior math ability students will be matched with average to low ability students. The pair/share system will also incorporate manipulatives and activities to accompany them. This will be done so that higher ability students will have the opportunity to reinforce their learning through teaching, and the lower ability students will receive extra tutoring from a peer. The personal dynamics of a peer allow for more hands-on activities to occur. Therefore, any manipulative tasks, such as measurement will insure high involvement for each member of the group.

Analytical Worksheet:

This tool is formulated for use with all mathematical operations and the categories

contained therein. Included on this worksheet will be a space for the facts of the problems a square for a simple sketch of the problem, a space for what they want to find out, a space for naming the operation, a space for identifying the category of the problem, and a box in which to write the answer.

The purpose of this worksheet is threefold: First, to guide the student in his/her reading comprehension of the problem. Second, to help the student clarify his/her interpretation of the text. Third, to increase the visual imagery while thinking about the problem and its solution.

Draw Pictures:

The teacher will model how to draw pictures/sketches of mathematical word problems. This will be taught as a separate lesson before distributing the analytical worksheet. Within this lesson the students will be shown the difference between a sketch and a drawing, andhow to label measurements of mass, volume, and length. In addition, students will be taught how to represent objects symbolically. This strategy will be used throughout intervention to promote the students' visual imagery abilities.

Math Journal:

The mathematics journal will he one of the devices that connects language arts skills to math skills. Within their journals, students will be able to express anxiety reflect upon mathematical concepts and communicate to the teacher problematic areas within the current unit. Divergent thinkers may use journal writing to find patterns within mathematical laws and theorems as a form of enrichment. Additionally, students may use the journal as a vehicle to express pride in their accomplishments.

Methods of Assessment

In order to assess the effects of the intervention, students will be grouped cooperatively and paired to solve real life word problems. In addition, students will be given analytical

worksheets, and a daily word problem in order for them to develop logical thought processes when encountering word problems. Within these aforementioned analytical worksheets students will learn how to represent word problems in the form of pictures when applicable. Furthermore, students will learn to differentiate between various types of word problems and the students will be taught strategies by which to address these word problem types. The analytical worksheet will assist the researcher by assessing whether these processes are adequately synthesized by the student. Another type of assessment will involve journaling. Students will be able to reflect upon the researchers strategies within a math journal. The researchers will analyze and assess these journals for math anxiety and comprehension of the techniques and strategies that the students are utilizing. The math journal anxiety tally sheet will be the means by which the researchers will accomplish this task. Finally, there will be a criterion based test that the students will be given before and after the intervention to assess its impact upon the students. This test will reflect the types of word problems that will be analyzed and classified by the students during the intervention. In addition, the test will be modified to suit the various grades that the researchers are assessing. To be specific, there will be a test for third, fifth, sixth, and eighth grade respectively.

CHAPTER FOUR

PROJECT RESULTS

Historical Description of Intervention

The objective of this project was to increase student confidence and ability when solving math word problems. Initially, the researchers implemented a 'problem of the day' in order to introduce students to various types of learning strategies when solving word problems. This intervention was consistent throughout the entire assessment period. Upon entering the classroom, students were challenged to solve a math word problem within a given time span. Later in the day, the researcher would discuss and teach appropriate strategy related to the morning word problem. For example, if 'working backwards' was the topic of the morning word problem, this strategy would be discussed and demonstrated later in the day. Problem types that were introduced to the 'problem of the day' strategy were meant to be classified over the length of the intervention.

The researchers initiated the 'classification of word problems' intervention by concentrating on one problem type per week, describing the similarities, word clues, and structural analysis of the problem type being considered. In conjunction with classifying word problem types, the researchers created analytical worksheets to be used by the students to further study the structure of a particular word problem. After the fifth week, the researchers agreed that

this strategy was too cumbersome and ineffective. However, the idea of introducing one problem type per week was effective to allow for adaptation of student thought processes. Therefore, problem classification on the researchers' part continued, but the students were no longer focused on the problem type, focusing rather on comprehension of the problem.

Five weeks into the intervention, students were able to describe and utilize strategies that were taught in the previous weeks. In addition to 'problem of the day,' cooperative groups and pair/share activities were used in order to provide students with both manipulatives and peer tutors to alleviate anxiety and bridge the gap between abstract and concrete mathematical concepts. These interventions were implemented twice a week after the fifth week of the assessment period to reinforce or further clarify abstract concepts.

The next intervention was math journals. This strategy also began during the first week of the intervention and was ongoing throughout the allotted time frame. At the end of each week students were instructed to write a math journal expressing comprehension, problem areas, concepts or activities that went well, and other thoughts on the math lessons relating to word problems. One of the researchers at site B increased the scope of the math journals to include additional days in which the students would answer situational type math problems in paragraph form relating to the concepts being taught at the time. This classroom utilized the math journal in two ways as opposed to merely being a tool by which to gage the students learning and/or frustration.

Lastly, it was the researchers intent to have the students sketch a picture to match each word problem type. This was to take place within the analytical worksheet as the means to further studying the structure of the word problem type involved. Once implementation of the intervention began the researchers realized as stated above that the analytical worksheet was too

cumbersome, repetitive, and was either over simplified or too complex for the word problem involved. One of the reasons for this was the fact that not all word problem types lend themselves to drawing a picture. Therefore, drawing a picture was only utilized when it was appropriate, or further enhanced comprehension of the word problem involved. This strategy of drawing a picture therefore, was implemented throughout the intervention as the need arose. It was also often used to clarify an idea or to help make terminology used in specific math problems clearer.

Upon reflection, some of the strategies used in our intervention may need to be modified or discarded. Some of the suggestions made by the research literature were not as effective as hoped, whereas some of the strategies worked well. The following is an explanation of the strategies implemented and their effectiveness in improving students' word problem solving abilities.

Our first strategy that was implemented into the classroom was to base our teaching of story problems on an outline of problem type classification. The word problems were divided into four basic operations. The problems under the operations were further classified according to the problems' characteristics. For example, within subtraction problems, one category would be comparison of quantities, (i.e. "David is 15 inches taller than Betty. Betty is 3'6", how tall is David"?) Another type of subtraction problem is an original number minus a quantity (i.e. "Alice has 26 cookies and hands out 9 of them. How many cookies does Alice have left?")

Some members of our research team found that using this matrix was not beneficial in increasing problem solving success. The primary students were too confused with its use. Some of the intermediate students felt it was a repetitive lesson since they already knew how to solve some of these problems. Furthermore, some of the poorer students were trying to memorize

patterns to solve the problems rather than understanding what the question was asking. For these students, the process prevented them from transferring the knowledge gained from the lessons as they sought instead to memorize steps that were not always applicable to the problem. Therefore, for some students, using a matrix was detrimental to the conceptualization of the story problem.

In contrast, two members of the research team found the matrix to be a useful guide when teaching story problems. The use of the matrix ensured that the researcher presented a wide array of story problem types. In this way, the researcher was assured the student had practiced a great variety of problem types. Without the use of the matrix as a guide, some of the different types of problems could have been overlooked within the daily lesson plans.

At the junior high level, students still used the matrix as a study guide when solving various problems. The students were able to refer to their notes in which the matrix was written to provide a reference when analyzing story problems.

The second strategy that we implemented in our research action plan was "the problem of the day." This intervention was the most effective strategy. The researchers felt that there were two main reasons as to why this strategy worked so well in the classroom. First, the use of the strategy seemed to alleviate some of the anxiety students face when solving problems. Students were more apt to finish and successfully complete one problem as opposed to several, which could be intimidating and result in failure and/or high frustration.

This intervention was consistent throughout the entire assessment period. Upon entering the classroom, students were challenged to solve a word problem within a given time span. Later in the day, the researcher would discuss and teach the appropriate strategy related to the morning word problem. Five weeks into the intervention, students were able to describe and utilize

strategies that were taught in the previous weeks. New concepts related to the intervention were then introduced on a weekly basis as opposed to a daily basis.

In addition, the students were able to assist each other and correct one another's mistakes, which added to their confidence and understanding of the concepts involved. Therefore, the students experienced a series of small successes which appeared to improve their overall attitudes toward the problem solving process. Furthermore, some teachers implemented this as an incentive program and gave rewards for correct answers. This further helped to improve their attitude toward solving problems.

Also at the junior high level, positive competitions were developed spontaneously as students tried to be among the first to solve the problems. Even low-achieving students would quickly get "on task" and seek assistance if necessary to successfully complete the problem. The researcher noticed a marked improvement in both time needed to solve problems and the level of accuracy in all ability levels as a result of their competition.

The second reason is closely related to the first in that time on task was increased because of the students' involvement and interests. Researchers indicated that one cause of failure when working problems was that students fail to spend enough time on trying to solve a problem. The success of the "problem of the day" intervention was partially based on students naturally increasing time spent on story problems as their interest increased. In addition, the 'problem of the day,' forced the teacher to incorporate minutes into each math class to the working of story problems. This resulted in an overall increase of time on task.

The third strategy, cooperative grouping, was also highly successful. First, this allowed students with poor reading skills to become involved in the problem solving process. Second, there was less pressure on the individual to be the only resource in solving a problem.

Furthermore, students who otherwise would still become overwhelmed, confused, or allow anxiety to frustrate them, now had the assistance of more capable math students to remediate and inspire them to utilize their creative abilities and fasten confidence while being taught the skills necessary for the lower achieving student to then complete similar problems on their own at a later date. Due to the occurrence of peer teaching and the confidence raising nature of cooperative grouping, the researcher was able to stimulate higher levels of thinking by providing more complicated and challenging problems than otherwise would be given to students in a traditional format classroom. Lastly, the researchers found that this mode of teaching was far more condusive to the use of manipulatives, which the recommended progression of instruction, in terms of concrete to abstract, states that this is essential for students to conceptualize and develop adequate problem solving skills.

Another strategy related to cooperative grouping was Pair/Share. This strategy contained similar benefits as to cooperative grouping but had some subtle differences. With only two in a group each student was forced into a higher level of involvement in the math activity. Furthermore, in a cooperative group weaker students could easily ride on the coat tails of a higher achiever, but in the pair share any student weaknesses could be more easily seen by the teacher. Therefore, the pair share strategy would be the recommended group dynamic to use when the teacher felt the students should be fairly familiar with a concept.

The fifth strategy was using an analytical worksheet which was unanimously eliminated by the research team after its use for one month. First, it was cumbersome because not all problems hit within the components of the worksheet. Secondly, some teachers found it tedious for the students to follow the process of filling in the worksheet, especially when the solution was recognized before the sheet was completed. Furthermore, researchers felt the use of the

worksheet actually redirected the focus of the students' thinking away from truly understanding a story problem. Instead students were thinking about the process and patterns involved in filling out the worksheet. Finally, it was a hassle to continue making copies of the worksheet.

The sixth strategy was to draw a picture when solving a problem. The consensus of the group was that illustrating the problems was a useful teaching tool. However, requiring its use in every problem was difficult because not all story problems are able to be illustrated. Teaching this as a strategy made students aware of this tool, as observation of students prior to the intervention did not make use of this tool. Post intervention, approximately 25% of the students were seen to be using this tool. Therefore, it was beneficial in terms of increasing the number of strategies at their disposal.

The seventh strategy was math journaling. The researchers would recommend the use of this intervention because of the following reasons: a) Students were able to express and reflect upon abstract mathematical concepts to further enhance their understanding of the mathematical processes involved in the real world as they relate to the operation or concept being taught, i.e., students were told to write about how you could use fractions in the real world; b) Students verbal linguistic intelligences were tapped to reinforce their mathematical/logical intelligences; c) students were allowed to express anxieties as well as discuss troublesome areas of understanding; d) finally, students were able to relate accomplishments and areas of growth. Both types of reflection allowed for the researchers to adapt lessons or monitor journals for understanding and attitudes.

Presentation and Analysis of Results

Before the intervention strategies were begun, students were given a teacher-made story problem test of ten to twelve questions. The same test was administered at the end of the

intervention time period. The following bar graphs compare the students' performance on the two assessments. The bar graphs compare the failure rate for each test question. Strategy use only was graded; computational errors were marked correct if proper strategy was used. The pie graphs compare the distribution of IOWA problem solving sub-test scores before and after the intervention.

Data Analysis – Site A, Classroom 1

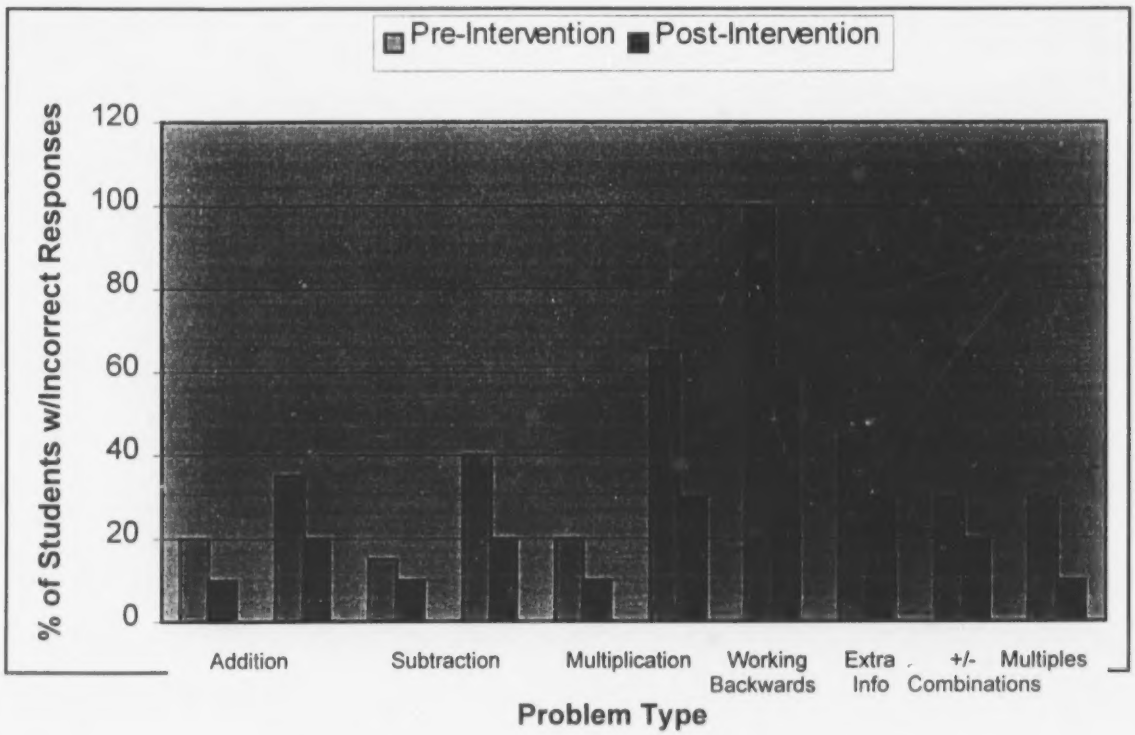


Figure 30. Comparison of Pre- and Post-Intervention test results (Site A, Classroom 1).

The graph above reflects the pre and post teacher-made test results. The students performed well in all areas with exception of the working backwards problem. In this area 60% of the students answered incorrectly. None of the other areas were over 30% incorrect post-intervention. All areas of the test improved by at least 10% with the exception of subtraction which already had a very low margin of error pre-intervention. Some explanations of this

increase in performance maybe related directly to the intervention techniques. However, some credit may lie in the increase of students abilities concerning all facets of math which they have been exposed to during the school year. Lastly, the post-intervention test which the students completed was identical, except for numbers, to the first pre-intervention test. This also may have aided the students performance on the second test.

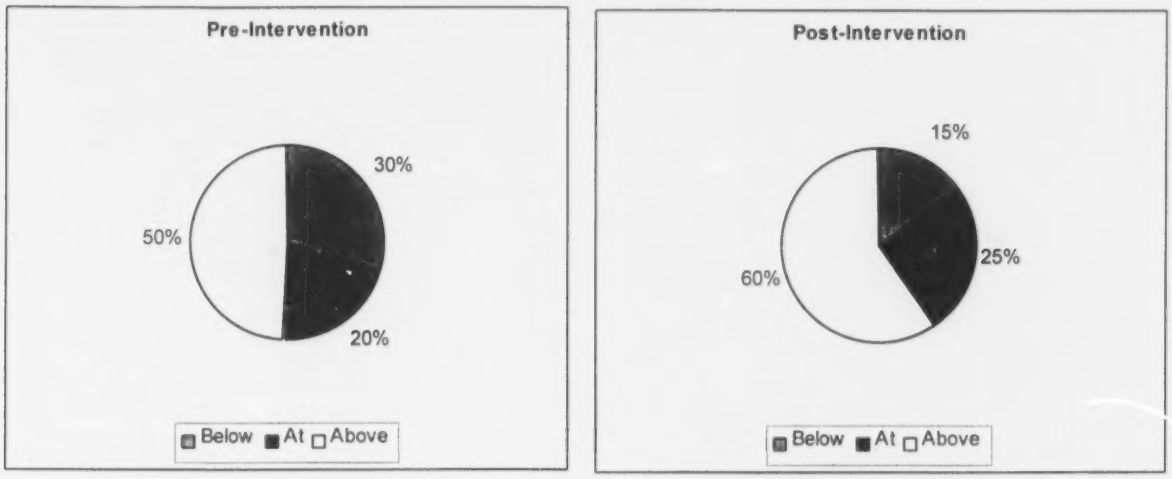


Figure 31..Percent of Class Above, At or Below Grade Level on IOWA Problem Solving Sub-test (Site A, Classroom 1).

The two graphs above reflect the Iowa scores exclusive to problem solving both pre and post-intervention. Students above grade level increased by 10% after the intervention. Students at grade level increased by 5% after the intervention. Students below grade level decreased by 15% after the intervention. This data would indicate that the intervention was effective as the students scores were not compared in terms of growth, but instead were compared in relation to their standings of last year at this time. For example, a student receiving a 5.6 in fifth grade and a 6.5 in sixth grade would be considered below grade level even though the student has shown a growth of eight months. Therefore, the increase of students above or at grade level is very

significant as in many cases the students grew more than a year or even a year and a half within a one years time frame.

Data Analysis – Site A, Classroom 2

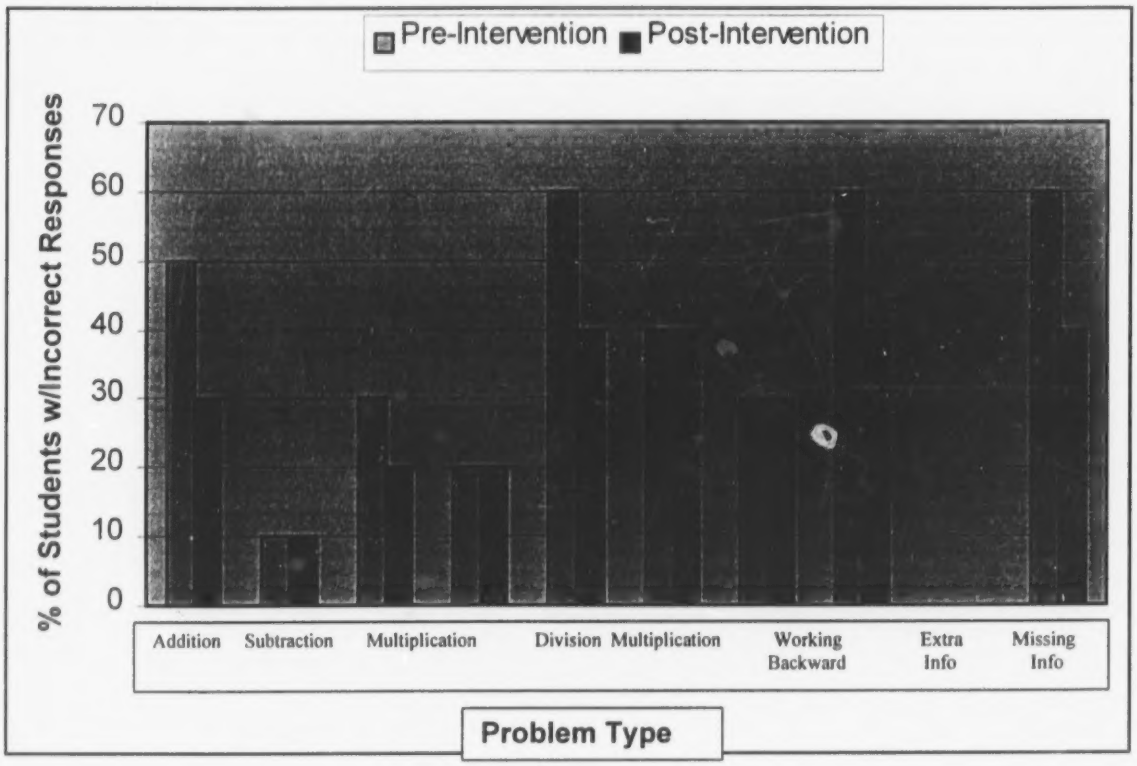


Figure 32. Comparison of pre- and post-intervention test results (Site A, Classroom 2).

Based on the post-intervention teacher-made test, students seemed to have learned how to apply strategies to some of the topics studied in math. While some score remained the same even after the intervention, none of the scores declined. Specifically, students' scores improved on problems involving two-step addition, one-step multiplication, one-step division, working backwards, and with problems with missing information. Students' scores remained the same as before the intervention on two-step subtraction, two and one-step multiplication, and on problems with extra information.

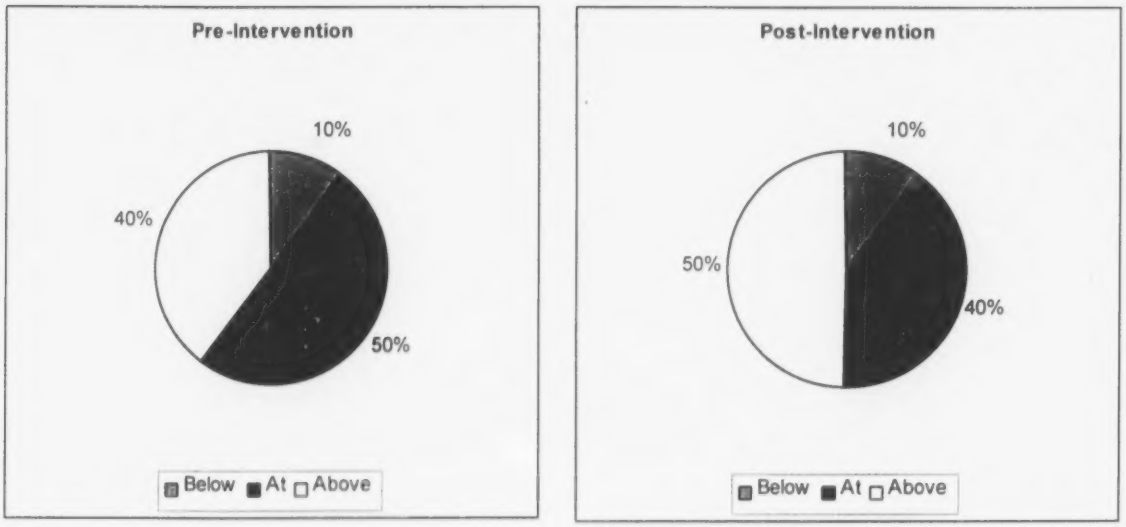


Figure 33. Percent of class above, at or below grade level on IOWA problem solving sub-test (Site A, Classroom 2).

Based on the post-intervention IOWA test of basic skills, students made slight improvements in solving multi-step math word problems. Last year, 50% of the student focus group were at grade level in terms of their scores on the IOWA test of basic skills. Forty percent of the group was above grade level, while 10% were below grade level. After the strategies were implemented, half of the group was above grade level on the IOWA test, while 40 % were at grade level and 10% remained below grade level.

Data Analysis – Site A, Classroom 3

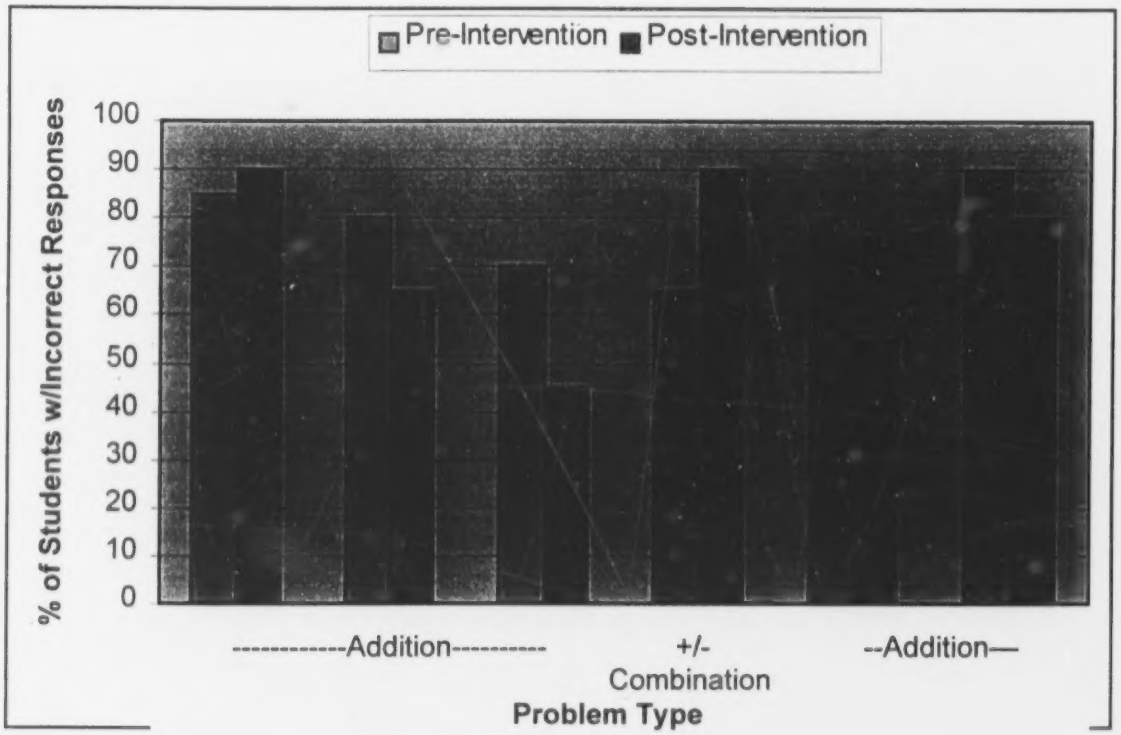


Figure 34. Comparison of Pre- and Post-Intervention Test Results (Problems 1 – 6) (Site A, Classroom 3).

The first test item was a one-step addition problem, which indicates that there was an increase of four percent who did poorly on the post assessment. The second test item, a two-step addition problem, showed an increase that 14% of the students improved on the second test. Item three, two-step addition, indicated improvement with l4 students missing this item on the first test and only 9 students missing it the second time. Item four, a two-step addition and subtraction, indicates that 26% more of the students incorrectly answered this problem. Items five, six, and seven indicate a positive growth in one and two-step addition and subtraction problems.

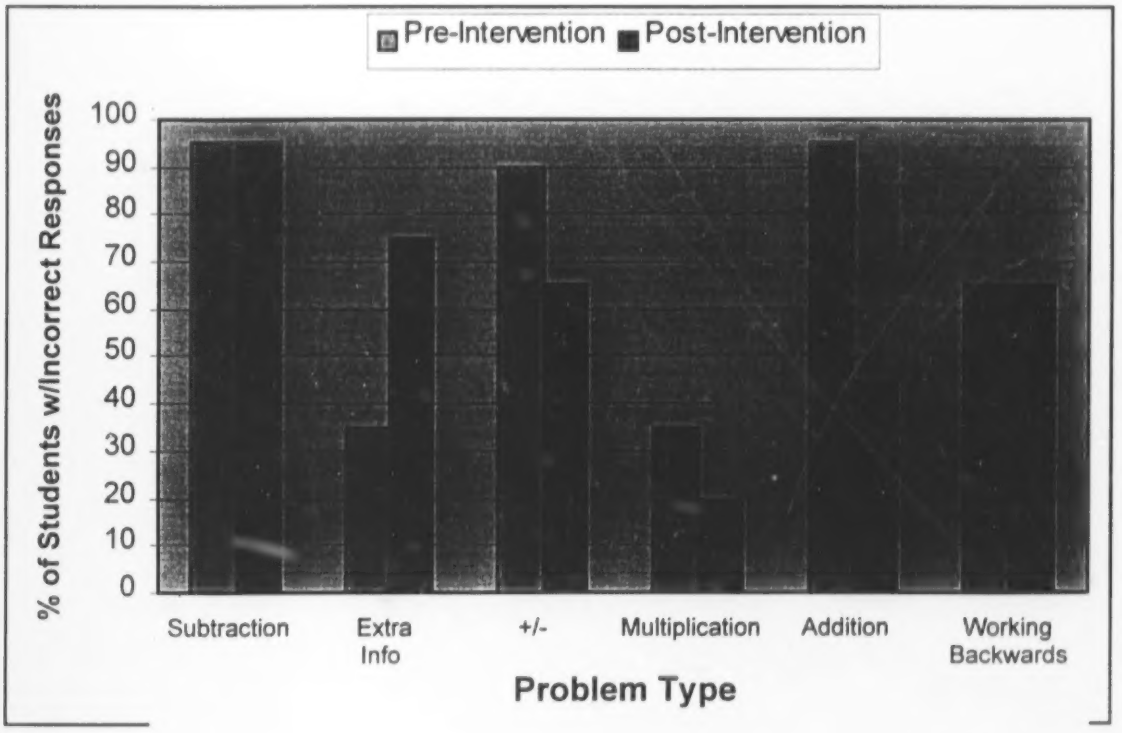


Figure 35. Comparison of Pre- and Post-Intervention Test Results (Problems 7 – 12) (Site A, Classroom 3).

Item eight of the assessment, extra information, showed the greatest increase of incorrect answers. Item nine, the combination of one-step addition and subtraction indicates a 24% improvement in solving the problem. Along with item ten, one-step multiplication showed a 12% improvement of correct answers. One of the greatest increases was item eleven, two-step addition, with a 48% drop in incorrect answers. Finally, the last test item, working backwards, the same number of students incorrectly responded on the post-intervention instrument as on the first. The teacher-made assessment indicated growth overall. Greater growth was indicated in the two-step problems because of adequate time on task given. Item one appeared to illustrate a lack of comprehension in problem solving. On items eight and twelve, students were given

insufficient time and exposure from both the researcher and the text. In addition, some variance in score ratio has occurred because eight of the original students included in the intervention have been transferred to a new school due to overcrowding.

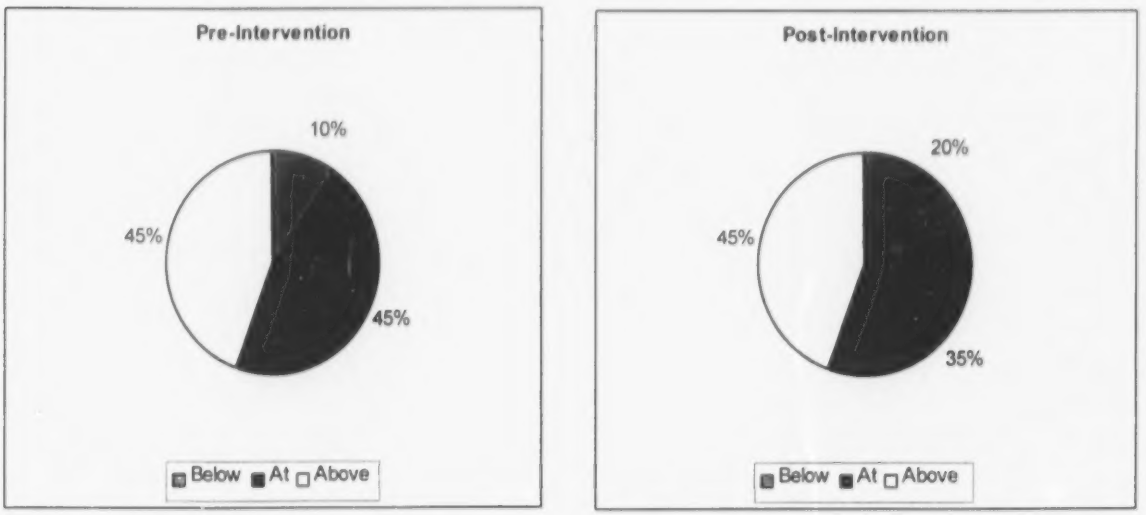


Figure 36. Percent of Class Above, At or Below Grade Level on IOWA Problem Solving Sub-test (Site A, Classroom 3).

The sub-test that students completed post-intervention shows that students above grade level remained the same. There was a slight increase for the students below level (10%) as well as proportionate amounts of loss occurring with students at grade level. As mentioned before, one of the reasons for this may be due to the change in number of students included in the intervention.

Data Analysis – Site B, Classroom 1

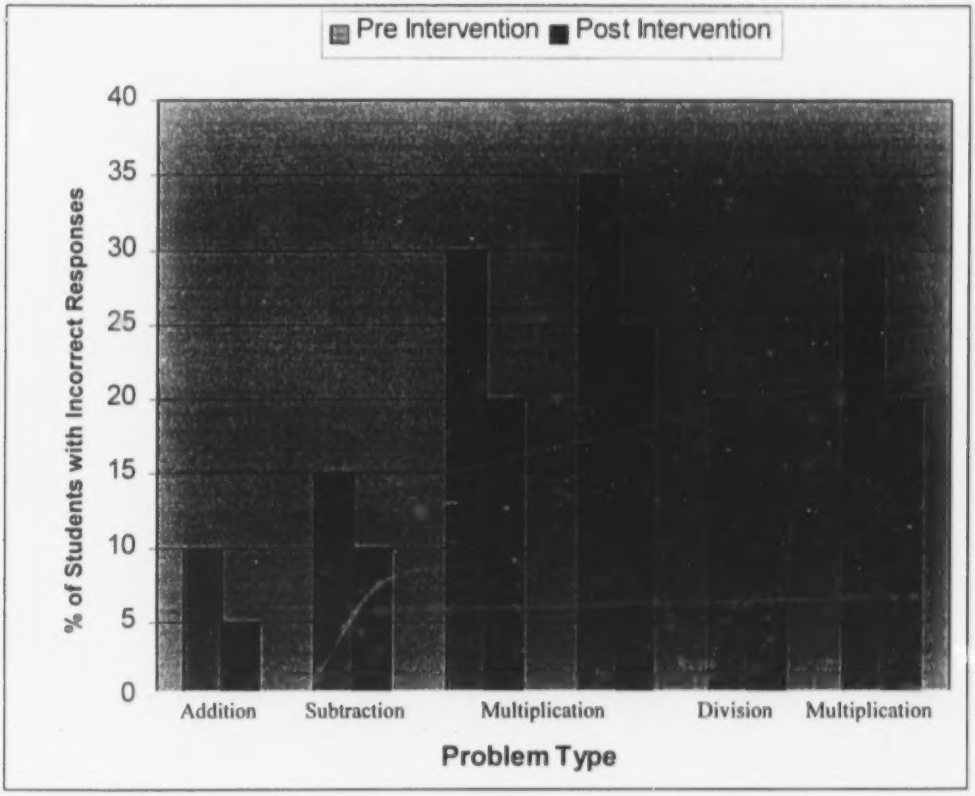


Figure 37. Comparison of Pre- and Post-Intervention Test Results (Problems 1 – 6) (Site B, Classroom 1).

The graph above reflects the pre and post teacher-made test questions one through six. The students showed a five to ten percent improvement in all question types. Addition, subtraction, and division showed the least margin of improvement. These areas however, already had a low margin of error, therefore, the improvement shown is proportional to the multiplication problems which had a five percent greater margin of improvement.

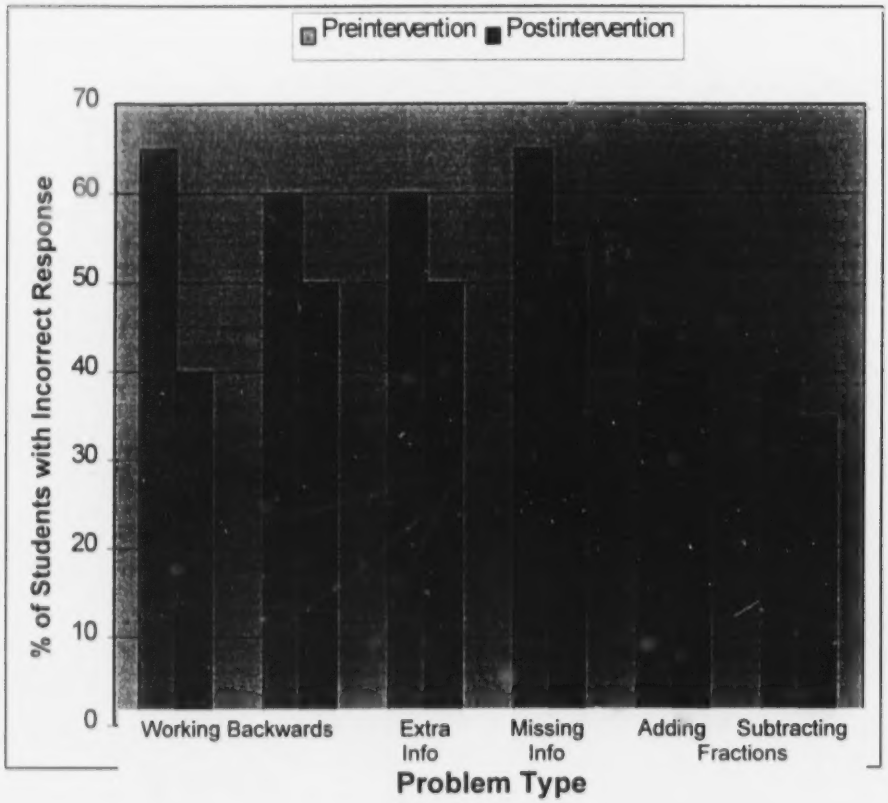


Figure 38. Comparison of Pre- and Post-Intervention Test Results (Problems 7 – 12) (Site B, Classroom 1).

The graph above reflects the pre and post teacher-made test results questions seven through twelve. Question seven shows the greatest margin of improvement – 25%. Perhaps this is due to the fact that most students indicated after the initial pre-test that they had no experience with a working backwards type problem. Even though this question exhibits the greatest margin of improvement there still is a 40% margin of error indicating that this remains the most difficult type of word problem for the students to solve. Problems nine and ten seem to reflect the same conclusions, that is difficult context with marginal improvements. The final two problems exhibited the least amount of improvement. Once again, post intervention margin of error was

minimal in numbers 11 and 12. In conclusion, when students exhibited the greatest amount of improvement it was in direct relation to the high percentage of initial incorrect responses. Therefore, the minimal improvements were just as significant, and in direct proportion to the areas of maximal improvements.

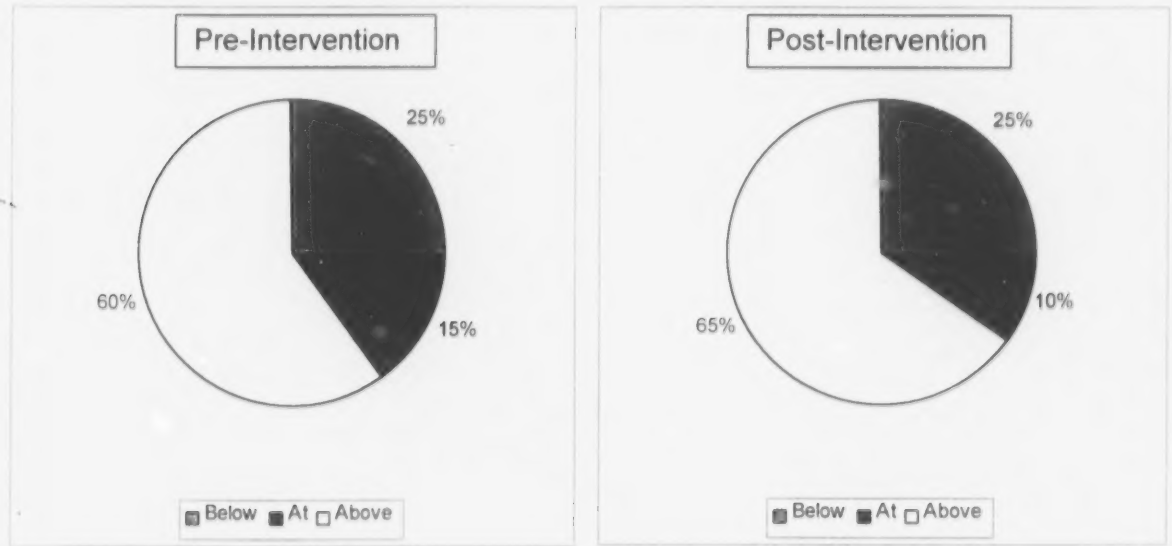


Figure 39. Percent of Class Above, At or Below Grade Level on IOWA Problem Solving Sub-test (Site B, Classroom 1).

These graphs reflect pre and post-intervention Iowa scores. Over all the scores remained rather static with the only significant gain being in students above grade-level. Below grade-level percentages remain the same and students at grade-level percentages were reduced from 15% to 10%. This loss however was shown as gain with students above grade-level. While this differential was slight it is significant as most students achieved more than one year's growth.

Data Analysis – Site B, Classroom 2

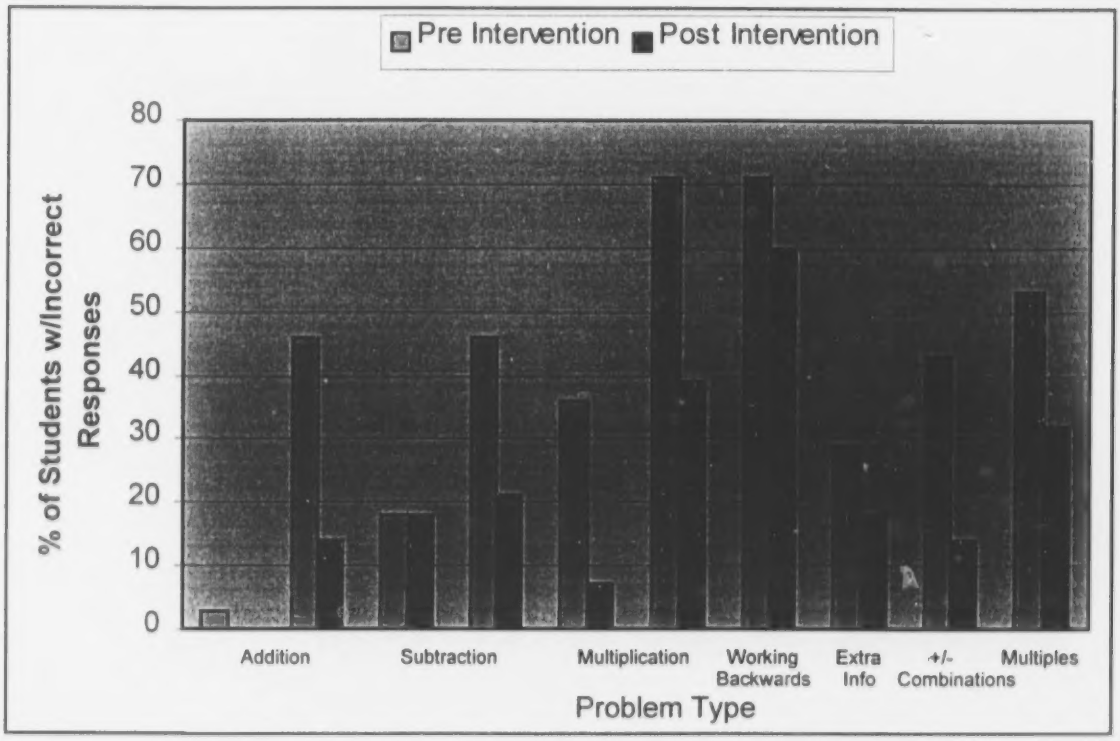


Figure 40. Comparison of Pre and Post-Intervention test results (Site B, Classroom 2).

The first test item, a single step addition problem, was correctly worked by virtually all students both times. The second item, a two step addition problem, indicated a more significant improvement with 13 students missing this item on the first test and only 4 students missing it on the second test. Item three, a one step subtraction, showed no change in number of incorrect answers. Item four, a two step subtraction problem, had 13 students miss it on the pre-intervention test while only 6 missed it on the post-intervention test. Likewise, item five, a one step multiplication problem, indicated positive growth with 10 students missing it on the pre-test and only 2 students missing it on the post-test. One of the greatest increases in score was on the two step multiplication problem, item six, with a drop of incorrect answers from 20 to 11. Item

seven, working backwards, and item eight, extra information, only showed a small change of three fewer wrong answers. Improvement was indicated on item nine, combination addition and subtraction, with a change of 12 incorrect answers to 4 incorrect answers. Finally, the last test item, which used multiples, had 15 incorrect responses pre-intervention and nine correct responses post-intervention.

The teacher-made test assessment indicated a growth in problem solving ability in general. The test score mean increased from 50 to 76; the median changed from 60 to 80; and the mode was raised from 70 to 90. Considering that this was a high-scoring class to begin with, the increase in these statistics is encouraging. An analysis of the test items reveals that there was greater growth in the two step problems. An explanation for this is that the class has a large number of above average students. Therefore, most of the one step problems were easy for them on the first test. Only a small number of students got these problems incorrect on the pre-test and no growth was shown in the data for this type of problem. However, there were more students who failed to solve the two step problems on the pre-test. Therefore the intervention strategies provided new knowledge for these kinds of problems.

Two test items deviated from expectations. First, item seven was a two step problem which showed little improvement from pre to post-test. An examination of the lesson plan schedule revealed this type of problem strategy, working backwards, had not been given adequate time on task. This would account for the continued high failure rate of 71% to 60%. Second, item three did not change at all, having five students failing to use a correct strategy. Strangely, only one of the original students who failed this item missed it again on the post-test. The other four students had done the same problem correctly the first time. The underlying explanation for this occurrence is beyond the scope of this research.

Another intervention assessment was a comparison of the students' scores on the IOWA Problem Solving Sub-test from the previous year to their scores on the same sub-test from a practice IOWA test given post-intervention. For this class the analysis of these scores was identical pre- and post-intervention: 23% below grade level; 4% at grade level; and, 73% above grade level. Since this class began with 73% above grade level, there was little possibility of change for the majority of the class. On the other hand, all of the students who were below grade level were the fifth graders who were chosen for placement in this class because of their low math scores. While the teacher feels that daily classroom behavior indicates growth in math understanding in this below level group, it has evidently not been enough to show up on the standardized test.

Conclusions

Based on the presentation and analysis of the data on math word problem strategies, student performances markedly improved in all aspects related to this topic. One of the areas in which students showed great improvement was in comprehending and discriminating between operations required to solve math word problems. The most useful tools in teaching this skill were cooperative grouping with manipulatives and the 'problem of the day.' In addition, the immersion of students into the concepts related to word problems in and of itself, developed within the students an innate ability to systematically solve problems based on their individual strengths and weaknesses. Within this process, students were not only able to assist peers in problem solving, but also to reap the benefits of step-by-step guidance of higher ability-level students.

Time-on-task was another area where improvement was illustrated within the classrooms. Through the use of journaling, students were able to express their anxieties and fuse language

arts skills with mathematical concepts. Furthermore, students were required to write their own math problems based on a given mathematical concept. This activity required divergent thinking to create the problems, and thus, brought about a realization of how the concepts relate to real-world scenarios. This resulted in an increased amount of time spent analyzing and solving word problems.

One improvement the researchers would make when implementing the interventions would be in the area of cooperative groups. While cooperative grouping worked well within introductory lessons, the effectiveness of this strategy dwindled as concepts became more complex. Within this scenario, a switch to pair/sharing allowed a greater participation on an individual basis, therefore increasing the comprehension of the processes involved within specific problems. Pair/sharing also insured that each student would interact directly with manipulatives. In larger groups, a portion of each group would not have the time nor the availability of manipulatives; therefore, these students would not have the same concrete experiences as other group members. Pair sharing seemed not only to alleviate the aforementioned dilemma, but when utilized in a heterogeneous manner provided other benefits to students as well. One of these benefits was that higher achieving students were able to further improve their problem solving abilities by having to teach the necessary steps to their peers. Lower or middle achieving students reaped the benefits of peer coaching as well as direct involvement with manipulatives.

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Appendices

Student Survey

Name _____

1) Do you like math? Why or why not.

2) Tell about a time in your life when you used math other than at school.

3) What is the most difficult part of math for you?

4) Which of the following do you dislike more? Circle one choice per letter.

A) multiplication or division

B) word problems or division

C) fractions or word problems
problems

D) multiplication or word

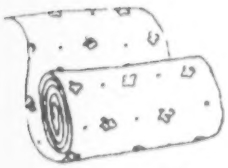
E) measuring or word problems

F) decimals or word problems

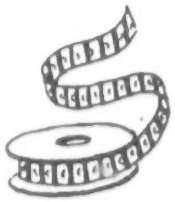
5) What do you think of when you first read a word problem? Be specific.

Name _____ Date _____

Use the price list to solve each problem.



fabric $3.98 yard



ribbon $1.50 yard

thread $1.79

buttons $1.25 pack

needles $1.98

scissors $4.59

A. Shelly buys 3 yards of fabric and a spool of thread. How much does she spend in all? _____

B. Sean buys 2 packs of buttons and a package of needles. How much does he spend altogether? _____

C. Yana buys 2 yards of ribbon. She gives the clerk $10.00. How much change does she get back? _____

D. Tony buys 3 pairs of scissors with a $20 bill. How much change will he get back? _____

Name _____ Date _____

E. Ethel buys 3 yards of ribbon and a pack of buttons. How much does she spend in all? _____

F. Tinfu buys 5 spools of thread and 1 pack of needles. How much does he spend in all? _____

G. Tasha buys 6 packs of needles. She gives the clerk $15.00. How much change does she get back? _____

H. Jamie bought candy for her birthday party. Six friends were coming and she bought 11 pieces of candy for each friend. She also bought three cupcakes for her brother and sister. How many pieces of candy did she buy? _____

I. Tony saved $1.50 to spend at Fun Time Park. Her mom gave her $6.00 more. It costs $4.50 to get into the park. How much will Tony have left? _____

J. Kyle bought 4 packs of cupcakes. There are 2 cupcakes in each pack. How many cupcakes did he buy in all? _____

K. Today Tina watered 67 trees, 85 bushes, and 248 pots of flowers. How many trees and bushes did she water in all? _____

L. Margie paid $50.00 for seven puppet show tickets. She got back $1.00 in change. How much did each ticket cost? _____ How did you find your answer?

Name _____ date _____

Math Story Problems

1) Mrs. White is buying new bedroom furniture. The new bed costs $210. The dresser costs $375. The last thing she picked out was a desk for $319. How much will all the furniture cost (not counting sales tax)?

2) Sara babysat on Tuesday and made $4.50. Then on Friday she made another $7.25. Mary says she made twice as much money as Sara. How much money did Mary make?

3) Mama Mio Pizza Company makes 736 pizzas a week. If 475 are cheese pizzas, how many are not cheese pizzas?

4) The grocery store put 120 boxes of corn flakes on the shelf. On Monday, 37 people bought on box each; on Tuesday, 21 customers bought corn flakes. How many are left on the shelf?

5) There are 14 rows of chairs in the theater. Each row has 9 seats. How many people can sit down?

6) Each lion in the zoo eats 86 pounds of meat in a week. If there are 8 lions, how much meat will they eat in 4 weeks?

7) Sam and Tom were trading baseball cards. First Sam gave Tom 10 cards. Then Tom gave Sam 16 cards. Now Sam has 71 cards. How many cards did Sam have before trading?

8) Mrs. Smith drove 12 miles on Tuesday. Mr. Black drove 17 miles on Wednesday. On Thursday, Mrs. Smith drives twice as far as Mr. Black did. How far did she go on Thursday?

9) Park School has 765 children. Keller School has 470 boys and 481 girls. How many more children does Keller School have?

10) Troy ate 15 pancakes. Barb ate twice as many. Betty ate twice as many as Barb. How many pancakes did Betty eat?

Name: _____ Date: _____
Solve.

1. Bill Johnson bought two adult tickets at $7.50 each, and four children's tickets at $3.75 each for a special children's performance. How much money did he spend for the tickets?

   _____

2. The circus purchases 500 kg of food for its animals each week. They pay $1.25 for each kilogram. How much did the food cost for a 4-week month? _____

3. Marci Hernandez bought 1 large balloon and *5* small balloons. The Large balloon cost $2.50. She spent a total of $6.25 What was the Cost of each small balloon? _____

4. Section A in the circus tent has 30 rows of seats with 20 seats in each Row. Section B has 20 rows with 25 seats in each row. Section C has 30 rows with 10 in each row. How many seats are there altogether? _____

5. The costume designer used 5.5 m of sequins for each costume use by the trapeze artists. If each of the five trapeze artists had four different costumes, how many meters of sequins were used altogether? _____

6. Judy ran for 3/4 h on Monday, 1/2 h on Wednesday, and 3/5 h on Friday. How many more hours did Judy run on Monday than on Friday? _____

7. Joey ran for 3/4 h on Monday, 1/2 h on Wednesday, and 3/5 h on Friday. flow many more hours did Joey run on Wednesday, than on Thursday? _____

8. Jim works for the Mountain Calculator Company. He is paid $3.80 an hour for the first 40 h and time and a half for overtime above 40 h. If Jim works 46 h during one week, how much will he earn? _____

9. A container of milk costs $.69 and a loaf of bread costs $1.15. How much will 5 containers of milk and 3 loaves of bread cost? _____

10. It takes 2 h of labor at $3.75 per hour and $2.50 in materials to mow your neighbor's lawn. What should you charge in order to make a $5 profit? _____

11 Jim had several pet rabbits. He gave 1/2 of them away and now has 3 rabbits. How many rabbits did Jim have to begin with? _____

12. Sharon has a certain amount of money in her checking account. She wrote checks for $42, $19, and $67. Sharon then had a balance of $225. How much did she have to begin with?

_____

Dear Teachers,

We would really appreciate your taking a few minutes to fill out this opinion survey about your students' math performance.

Chose one answer.

1) Most students perform better on math

a) ____ computation problems b) ____ story problems

2) It seems that students taking the IOWA generally do better on the math test involving

a) ____ computation problems b) ____ story problems

3) The most difficult math lessons to teach involve

a) ____ computation problems b) ____ story problems

c) other _____

Please circle SD for Strongly Disagree, D for Disagree, A for Agree & SA for Strongly Agree.

My students have trouble with word problems because of

4) Poor reading skills	SD	D	A	SA
5) Lack of strategy use	SD	D	A	SA
6) Lack of self confidence	SD	D	A	SA
7) A "let's just get an answer." attitude.	SD	D	A	SA
8) Insufficient time on task	SD	D	A	SA

9) I devote time to explicitly teaching word problem strategies

____ daily

____ weekly

____ bi-monthly

____ monthly

____ as the math text indicates

Thank you *So Much!*

Appendix F
WORD PROBLEMS FOR DIVISION

Measurement Problems

These first examples correspond to the rate-times-quantity multiplication problems. The quantity (number of sets) is the unknown, while the total and the rate (size of set) is given.

Mark has 24 apples. He put them into bags of 6 apples each. How many bags did Mark use?

Jill bought apples for 7 cents a piece. The total cost of her apples was 35 cents. How many apples did Jill buy?

Peter walked 12 miles at a rate of 4 miles per hour. How many hours did it take Peter to walk the 12 miles.

The next two examples are modeled in a similar manner but correspond to the multiple-times-a-quantity problems. Here the multiplier is unknown. See if you can see how these are the same and how they are a little different from the examples just given.

Mark picked 24 apples and Jill picked only 6. How many times as many apples did Mark pick than Jill did?

This year Mark saved $24. Last year he saved $6. How many times as much money did he save this year over last year?

Partition Problems

These first partition problems correspond to the rate-times-quantity problems with the rate (size of set) the unknown and the quantity (number of sets) and total given.

Mark has 24 apples. He wants to share them equally among his 4 friends. How many apples will each friend receive?

Jill paid 35 cents for 5 apples. What was the cost of 1 apple?

Peter walked 12 miles in 3 hours. How many miles per hour (how fast) did Peter walk?

The second and third examples are more difficult because price ratios and rates of speed are more difficult for children to understand and model. At the upper grades, progress can be made with rate problems like these by using models to help children see how they are like the easier sharing and repeated-subtraction problems.

The next two problems involve an unknown quantity (set size) with a given multiplier of the set and a total. Again, you are encouraged to see how these problems are alike and different from the examples just given and to relate them to the multiplication examples.

Mark picked 24 apples. He picked 4 times as many apples as Jill. How many apples did Jill pick?

This year Mark saved 4 times as much money as he did last year. If he saved $24 this year, how much did he save last year?

Remainders in Word Problems

Earlier (Figure 6.28) it was noted that when division problems do not come out evenly, the remainder is either "left over" or can be partitioned to form a fraction. In real contexts, remainders sometimes have three additional effects on answers: (1) the remainder is discarded, leaving a smaller whole number answer, (2) the remainder can "force" the answer to the next highest whole number, and (3) the answer is rounded to the nearest whole number for an approximate result. The following problems illustrate all five possibilities.

You have 30 pieces of candy to share fairly with 7 children. How many will each receive? Ans.: 4 and 2 left over. (left over)

Each jar holds 8 ounces of liquid. If there are 46 ounces in the pitcher, how many jars will that be? Ans.: 5 and 6/8 jars. (partitioned as a fraction)

The rope is 25 feet long. How many 7-foot jump ropes can be made? Ans.: 3. (discarded)

The ferry can hold 8 cars. How many trips will it have to make to carry 25 cars across the river? Ans.: 4. (forced to next whole number)

Six children are planning to share a bag of 50 pieces of bubble gum. About how many will each get? Ans.: about 8. (rounded, approximate result)

Students should not just think of remainders as "R3" or left over. They should be put in context and dealt with accordingly.

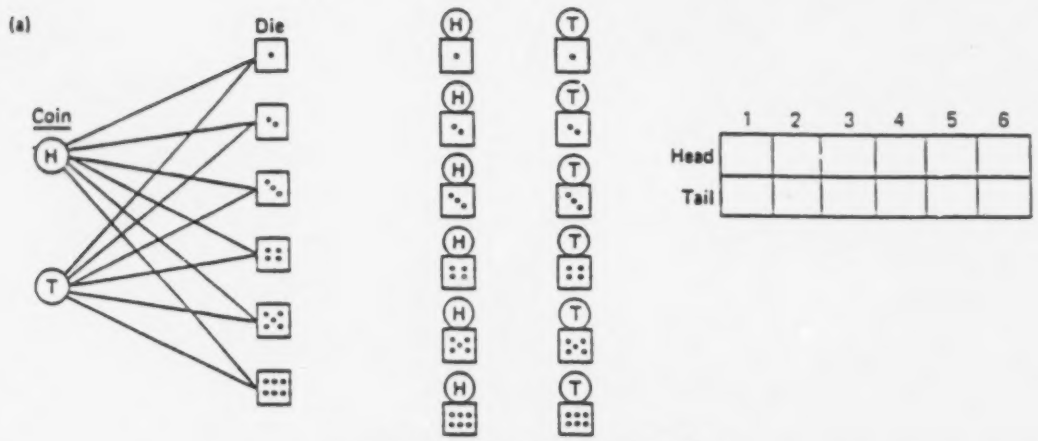
3). **Combination Problems (A Quantity Times a Quantity)**

Sam Slick is really excited about his new clothing purchases. He bought 4 pairs of pants and 3 jackets, and they all can be mixed or matched. For how many days can Sam wear a different outfit if he wears one new pair of pants and one new jacket each day?

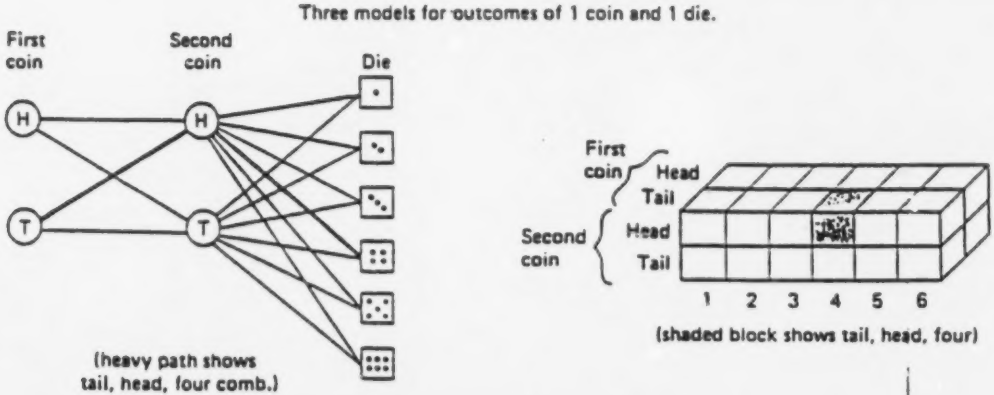
You want to make a set of attribute pieces that have 3 colors and 6 different shapes. If you want your set to have exactly 1 piece for every possible combination of shapes and colors, how many pieces will you need to make?

An experiment involves tossing a coin and rolling a die. How many different possible results or outcomes can this experiment have?

These problems reflect directly the combination concept of multiplication. There are different ways to model these problems: indicate pairings using lines, model all pairs directly, or use an array. Figure 6.24(a) shows how the coin and die experiment could be modeled all three ways.



Three models for outcomes of 1 coin and 1 die.



Two possible models for outcomes of two coins and one die.

Figure 6.24
Two possible models for outcomes of 2 coins and 1 die. Modeling outcomes of simple experiments.

Appendix F

WORD PROBLEMS FOR MULTIPLICATION

1). Rate Times a Quantity

In each of the following, one of the numbers is a *rate* or comparison of some amount to one set or one item. Rate examples include the "number in each set," the "cost per unit item," or the "speed per unit time." The second number is an actual *quantity*. It is the number of sets or the number of units to which the rate applies.

Mark has 4 bags of apples. There are 6 apples in each bag. How many apples does Mark have altogether?

Apples cost 7 cents each. Jill bought 4 apples. How much did they cost in all?

Peter can walk 4 miles per hour. If he walks at that rate for 3 hours, how far will he have walked?

In Figure 6.23, each problem is modeled with sets, an array, and a number line. An examination of the models will show how these three problems are alike. In the modeled form, the rate can be seen as the number in each set or the size of the set, and the quantity is the number of the sets. It is difficult for children to deal with price and time rates in problems. Helping them translate these ideas to models can help them relate these ideas to the more basic equal-addition concept they have developed for multiplication.

2). Multiples of a Quantity

In these problems only one number is an actual quantity and stands as a reference set. The other number is a *multiplier* that indicates how many copies of the reference are in the total or product. These problems can be interpreted using any of the repeated-addition models.

Jill picked 6 apples. Mark picked 4 times as many apples as Jill. How many apples did Mark pick?

Mark now has 4 times as many dollars saved as he had last year. Last year he had $6. How many dollars does he have now?

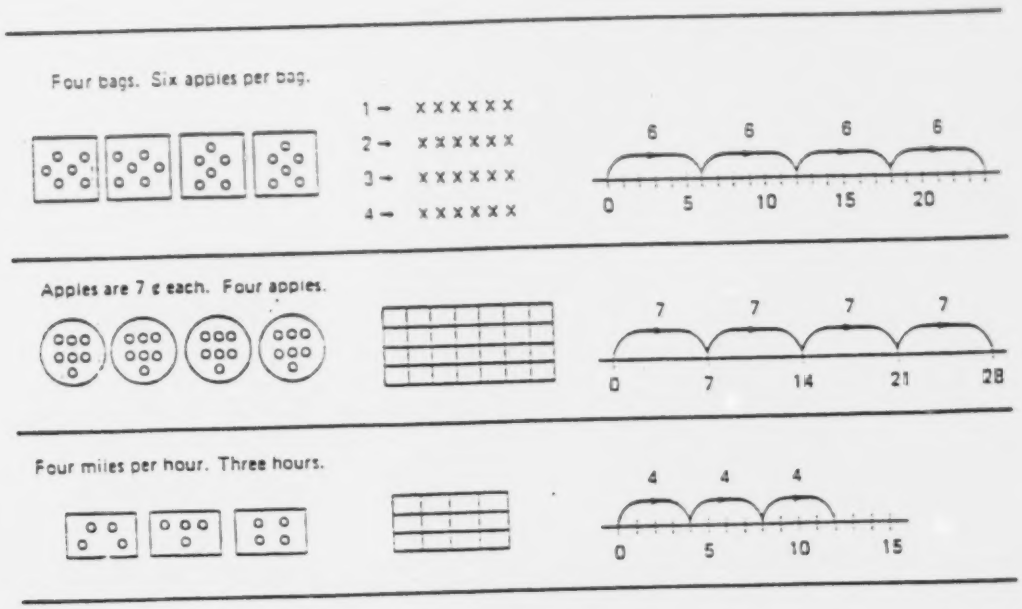


Figure 6.23
Three models of three rate times quantity problems.

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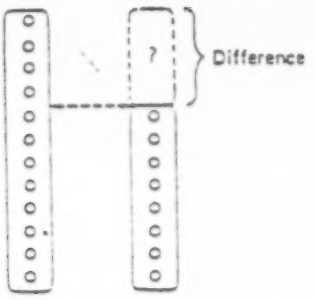
3). Compare Problems

There are three types of compare problems corresponding to which quantity is unknown (smaller, larger, or difference). For each of these, two examples are given: one in terms of more, the other in terms of less.

Compare: Difference unknown.

George has 12 pennies and Sandra has 8 pennies. How many more pennies does George have than Sandra?

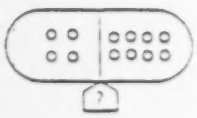
George has 12 pennies. Sandra has 8 pennies. How many fewer pennies does Sandra have than George?



2). Part-Part-Whole Problems

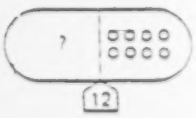
Part-part-whole: Whole unknown.

George has 4 pennies and 8 nickels. How many coins does he have?



Part-part-whole: Part unknown.

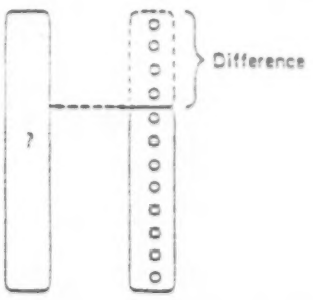
George has 12 coins. Eight of his coins are pennies, and the rest are nickels. How many nickels does George have?



Compare: Larger unknown.

George has 4 more pennies than Sandra. Sandra has 8 pennies. How many pennies does George have?

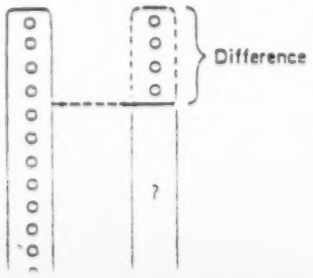
Sandra has 4 fewer pennies than George. Sandra has 8 pennies. How many pennies does George have?



Compare: Smaller unknown.

Sandra has 4 fewer (less) pennies than George. George has 12 pennies. How many pennies does Sandra have?

George has 4 more pennies than Sandra. George has 12 pennies. How many pennies does Sandra have?



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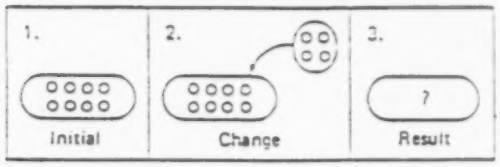
WORD PROBLEMS FOR ADDITION AND SUBTRACTION

Join and Separate Problems

For each of the two actions of join and separate, there are three quantities involved: the *initial* quantity, the *change* quantity (amount joined or removed), and the *resulting* quantity. Any one of the three quantities can be the unknown. An example of each of the six possibilities (three for join, three for separate) is given here.

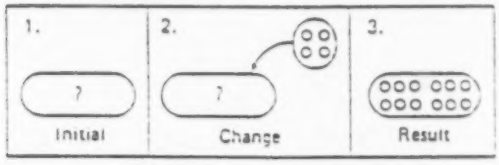
Join: Result unknown

Sandra had 8 pennies. George gave her 4 more. How many pennies does Sandra have altogether?



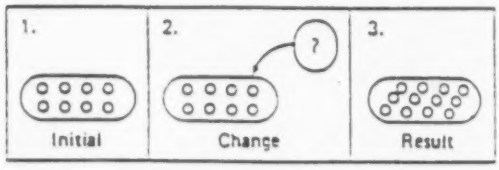
Join: Initial unknown.

Sandra had some pennies. George gave her 4 more. Now Sandra has 12 pennies. How many pennies did Sandra have to begin with?



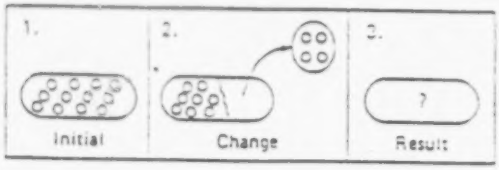
Join: Change unknown.

Sandra had 8 pennies. George gave her some more. Now Sandra has 12 pennies. How many did George give her?



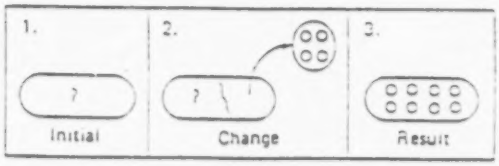
Separate: Result unknown.

Sandra had 12 pennies. She gave 4 pennies to George. How many pennies does Sandra have now?



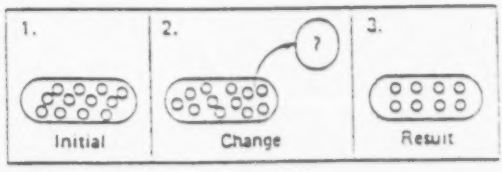
Separate: Initial unknown.

Sandra had some pennies. She gave 4 to George. Now Sandra has 8 pennies left. How many pennies did Sandra have to begin with?



Separate: Change unknown.

Sandra had 12 pennies. She gave some to George. Now Sandra has 8 pennies. How many did she give to George?



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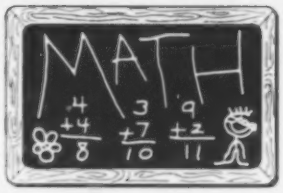
Name _____

Story Problem Worksheet



1) Facts:

2) Draw a picture of the problem.

3) What is the question?

4) What kind of problem is this? _____

5) Solve:

6) Answer